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Research Article

Stability and Convergence Analysis for Set-Valued Extended Generalized Nonlinear Mixed Variational Inequality Problems and Generalized Resolvent Dynamical Systems

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In this paper, we study a set-valued extended generalized nonlinear mixed variational inequality problem and its generalized resolvent dynamical system. A three-step iterative algorithm is constructed for solving set-valued extended generalized nonlinear variational inequality problem. Convergence and stability analysis are also discussed. We have shown the globally exponential convergence of generalized resolvent dynamical system to a unique solution of set-valued extended generalized nonlinear mixed variational inequality problem. In support of our main result, we provide a numerical example with convergence graphs and computation tables. For illustration, a comparison of our three-step iterative algorithm with Ishikawa-type algorithm and Mann-type algorithm is shown.

1. Introduction

Variational inequality theory was introduced by Hartmann and Stampacchia [1] in 1966 as a tool for the study of partial differential equations with applications principally drawn from mechanics. Variational inclusions are the generalized forms of variational inequalities and they have wide range of applications in industry, mathematical finance, and economics and in several branches of applied sciences; see [2–6] and the references therein.

We would like to emphasize that the projection method cannot be applied for solving the mixed variational inequalities involving the nonlinear term. To overcome this drawback, it is assumed that the nonlinear term involved in the general mixed variational inequalities is a proper, convex, and lower-semicontinuous functional. It is well known that the subdifferential of a proper, convex, and lower-semicontinuous functional is a maximal monotone operator. This characterization enables researchers to define the resolvent operator associated with the maximal monotone operator; see, for example, [7–9] and the references therein. The resolvent operator technique is used to establish the equivalence between the variational inequalities and fixed-point problems; see, for example, [10–12] and the references therein.

In recent years, considerable interest has been shown in developing various extensions and generalizations of variational inequalities and variational inclusions, both for their own sake and for their applications; see, for example, [13–16] and the references therein. There are significant developments of these problems related to set-valued operators, nonconvex optimization, iterative methods, and structural analysis. Glowinski and Tallec [17] suggested and analyzed some three-step splitting methods for solving variational inequality problems by using the Lagrange multipliers technique. They showed that three-step splitting methods