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Variational-Like Inequality Problem Involving Generalized Cayley Operator

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Abstract: This article deals with the study of a variational-like inequality problem which involves the generalized Cayley operator. We compare our problem with a fixed point equation, and based on it we construct an iterative algorithm to obtain the solution of our problem. Convergence analysis as well as stability analysis are studied.

Keywords: algorithm; solution; stability; sequence; inclusion

Citation: Rather, Z.A.; Ahmad, R.; Wen, C.-F. Variational-Like Inequality Problem Involving Generalized Cayley Operator. *Axioms* **2021**, *10*, 133. https://doi.org/10.3390/axioms10030133

Academic Editor: Wei-Shih Du

Received: 1 June 2021
Accepted: 24 June 2021
Published: 26 June 2021

1. Introduction

The mathematical formalism of a classical variational inequality problem is to find $y \in \mathcal{H}$ such that

$$\langle Ty, x - y \rangle \geq 0, \ \forall\, x \in \mathcal{H}, \tag{1}$$

where $\mathcal{H}$ is a Hilbert space and $T : \mathcal{H} \to \mathcal{H}$ is a nonlinear operator. The concept of variational inequalities was introduced by Stampacchia [1] and Fichera [2], separately. The variational inequality theory has received adequate recognition due to its implementation in a diverse range of problems arising in economics, physics, mathematical finance, structural analysis and in many branches of social, pure and applied sciences, see, for example, in [3–14]. Stampacchia [1] proved that the possible problems related with elliptic equations can be analysed through variational inequalities. Combining auxiliary principle technique and projection operator technique, Lions and Stampacchia [15] studied the existence of solution of variational inequalities.

The variational-like inequalities are the generalized forms of variational inequalities and provide us cogent tools to study many problems of basic and applied sciences. It is obvious that variational inequalities and variational-like inequalities are analogous of fixed point equations. This flipside equivalent formulation plays a significant role in many aspects of variational inequalities and variational-like inequalities. More precisely, this equivalent formulation is used to develop iterative algorithms and to study numerical methods related to variational inequalities and variational-like inequalities, etc.

It is well known that Cayley transform is a mapping between skew-symmetric matrices and special orthogonal matrices. As far as Hilbert spaces are concerned, Cayley transform is a mapping between linear operators. This transform is a homography having applications in real analysis, complex analysis and quaternionic analysis, etc.

As this subject is application oriented, in this paper, we consider a variational-like inequality problem involving generalized Cayley operator. An iterative algorithm is defined to obtain the solution of variational-like inequality problem involving generalized Cayley operator. For more details and recent developments of the subject, we refer to the