

Research Article

Generalized Complementarity Problem with Three Classes of Generalized Variational Inequalities Involving \oplus Operation

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Received 11 November 2020; Accepted 8 January 2021; Published 25 January 2021

Academic Editor: Sun Young Cho

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In this study, we introduce and study a generalized complementarity problem involving XOR operation and three classes of generalized variational inequalities involving XOR operation. Under certain appropriate conditions, we establish equivalence between them. An iterative algorithm is defined for solving one of the three generalized variational inequalities involving XOR operation. Finally, an existence and convergence result is proved, supported by an example.

1. Introduction

It is well known that the many unrelated free boundary value problems related to mathematical and engineering sciences can be solved by using the techniques of variational inequalities. In a variational inequality formulation, the location of the free boundary becomes an intrinsic part of the solution, and no special devices are needed to locate it. Complementarity theory is an equally important area of operations research and application oriented. The linear as well as nonlinear programs can be distinguished by a family of complementarity problems. The complementarity theory have been elongated for the purpose of studying several classes of problems occurring in fluid flow through porous media, economics, financial mathematics, machine learning, optimization, and transportation equilibrium, for example, [1–5].

The correlations between the variational inequality problem and complementarity problem were recognized by Lions [6] and Mancino and Stampacchia [7]. However, Karamardian [8, 9] showed that both the problems are equivalent if the convex set involved is a convex cone. For more details on variational inequalities and complementarity problems, refer to [6, 10–12].

The exclusive “XOR,” sometimes also exclusive disjunction (short: XOR) or antivalence, is a Boolean operation

which only outputs true if only exactly one of its both inputs is true (so, if both inputs differ). There are many applications of XOR terminology, that is, it is used in cryptography, gray codes, parity, and CRC checks. Commonly, the \oplus symbol is used to denote the XOR operation. Some problems related to variational inclusions involving XOR operation were studied by [13–16].

Influenced by the applications of all the above discussed concepts in this study, we introduce and study a generalized complementarity problem involving XOR operation with three classes of generalized variational inequalities involving XOR operation. Some equivalence relations are established between them. An existence and convergence result is proved for one of the three types of generalized variational inequalities involving XOR operation. For illustration, an example is provided.

2. Some Basic Concepts and Formulation of the Problem

Throughout this study, we assume E to be real ordered Banach space with norm $\|\cdot\|$ and E^* be its dual space. Suppose that d is the metric induced by the norm, 2^E (respectively, $CB(E)$) is the family of nonempty (respectively,