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Statistical Inference and Goodness-of-Fit Assessment Using the AAP-X Probability Framework with Symmetric and Asymmetric Properties: Applications to Medical and Reliability Data

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Abstract: Probability models are instrumental in a wide range of applications by being able to accurately model real-world data. Over time, numerous probability models have been developed and applied in practical scenarios. This study introduces the AAP-X family of distributions—a novel, flexible framework for continuous data analysis named after authors Aadil Ajaz and Parvaiz. The proposed family effectively accommodates both symmetric and asymmetric characteristics through its shape-controlling parameter, an essential feature for capturing diverse data patterns. A specific subclass of this family, termed the "AAP Exponential" (AAPEx) model is designed to address the inflexibility of classical exponential distributions by accommodating versatile hazard rate patterns, including increasing, decreasing and bathtub-shaped patterns. Several fundamental mathematical characteristics of the introduced family are derived. The model parameters are estimated using six frequentist estimation approaches, including maximum likelihood, Cramer-von Mises, maximum product of spacing, ordinary least squares, weighted least squares and Anderson–Darling estimation. Monte Carlo simulations demonstrate the finitesample performance of these estimators, revealing that maximum likelihood estimation and maximum product of spacing estimation exhibit superior accuracy, with bias and mean squared error decreasing systematically as the sample sizes increases. The practical utility and symmetric-asymmetric adaptability of the AAPEx model are validated through five real-world applications, with special emphasis on cancer survival times, COVID-19 mortality rates and reliability data. The findings indicate that the AAPEx model outperforms established competitors based on goodness-of-fit metrics such as the Akaike Information Criteria (AIC), Schwartz Information Criteria (SIC), Akaike Information Criteria Corrected (AICC), Hannan–Quinn Information Criteria (HQIC), Anderson–Darling (A*) test statistic, Cramer-von Mises (W*) test statistic and the Kolmogorov-Smirnov (KS) test statistic and its associated *p*-value. These results highlight the relevance of symmetry in real-life data modeling and establish the AAPEx family as a powerful tool for analyzing complex data structures in public health, engineering and epidemiology.



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