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A Robust Framework for Probability Distribution Generation: Analyzing Structural Properties and Applications in Engineering and Medicine

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Abstract: This study introduces a novel trigonometric-based family of distributions for modeling continuous data through a newly proposed framework known as the ASP family, where 'ASP' represents the initials of the authors Aadil, Shamshad, and Parvaiz. A specific subclass of this family, termed the "ASP Rayleigh distribution" (ASPRD), is introduced that features two parameters. We conducted a comprehensive statistical analysis of the ASPRD, exploring its key properties and demonstrating its superior adaptability. The model parameters are estimated using four classical estimation methods: maximum likelihood estimation (MLE), least squares estimation (LSE), weighted least squares estimation (WLSE), and maximum product of spaces estimation (MPSE). Extensive simulation studies confirm these estimation techniques' robustness, showing that biases, mean squared errors, and root mean squared errors consistently decrease as sample sizes increase. To further validate its applicability, we employ ASPRD on three real-world engineering datasets, showcasing its effectiveness in modeling complex data structures. This work not only strengthens the theoretical framework of probability distributions but also provides valuable tools for practical applications, paving the way for future advancements in statistical modeling.

Keywords: ASP transformation; Rayleigh distribution; moments; entropy; order statistics; maximum likelihood estimation

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1. Introduction

In the past few decades, the increasing complexity of real-world data has motivated researchers to develop more advanced probability distributions. While traditional models form the foundation of statistical analysis, they often lack the necessary flexibility and accuracy for modern applications. To overcome these limitations, several generalized families of distributions have been introduced, extending classical models to enhance both theoretical insights and practical applications.