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Advanced Lifetime Modeling Through APSR-X Family with Symmetry Considerations: Applications to Economic, Engineering and Medical Data

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Abstract

This paper introduces a novel and flexible class of continuous probability distributions, termed the Alpha Power Survival Ratio-X (APSR-X) family. Unlike many existing transformation-based families, the APSR-X class integrates an alpha power transformation with a survival ratio structure, offering a new mechanism for enhancing shape flexibility while maintaining mathematical tractability. This construction enables fine control over both the tail behavior and the symmetry properties, distinguishing it from traditional alpha power or survival-based extensions. We focus on a key member of this family, the two-parameter Alpha Power Survival Ratio Exponential (APSR-Exp) distribution, deriving essential mathematical properties including moments, quantile functions and hazard rate structures. We estimate the model parameters using eight frequentist methods: the maximum likelihood (MLE), maximum product of spacings (MPSE), least squares (LSE), weighted least squares (WLSE), Anderson–Darling (ADE), right-tailed Anderson–Darling (RADE), Cramér–von Mises (CVME) and percentile (PCE) estimation. Through comprehensive Monte Carlo simulations, we evaluate the estimator performance using bias, mean squared error and mean relative error metrics. The proposed APSR-X framework uniquely enables preservation or controlled modification of the symmetry in probability density and hazard rate functions via its shape parameter. This capability is particularly valuable in reliability and survival analyses, where symmetric patterns represent balanced risk profiles while asymmetric shapes capture skewed failure behaviors. We demonstrate the practical utility of the APSR-Exp model through three real-world applications: economic (tax revenue durations), engineering (mechanical repair times) and medical (infection durations) datasets. In all cases, the proposed model achieves a superior fit over that of the conventional alternatives, supported by goodness-of-fit statistics and visual diagnostics. These findings establish the APSR-X family as a unique, symmetry-aware modeling framework for complex lifetime data.

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