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Research Article

A novel probability distribution for count data modelling based on Bernoulli–Poisson-Ailamujia convolution: properties, regression framework and applications

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Abstract

The convolution of independent random variables serves as a powerful methodological framework for developing flexible probability distributions. This study uses this methodology to introduce the Bernoulli-Poisson-Ailamujia distribution. The new model can accommodate both overdispersed and underdispersed count data simultaneously. We derive various statistical properties of the new distribution, including moments, index of dispersion, mode, skewness, and kurtosis. We also study parameter estimation using maximum likelihood estimation and assess the finite-sample behavior of the estimators using Monte Carlo simulations. Furthermore, we also develop a flexible count regression model based on the Bernoulli-Poisson-Ailamujia distribution to investigate the relationship between covariates and count responses. Applications of the new model to three real-world datasets demonstrate its superiority over competing models.

Mathematics Subject Classification (2020). 62E15, 62F10, 62F12, 60E05

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1. Introduction

Count data are commonly used in various scientific and industrial disciplines, such as medicine, finance, and engineering. Due to its complex nature, modelling count data has gained significant attention over the years. Although the Poisson distribution has been a standard choice for this data type, its reliance on the restrictive assumption of mean-variance equality makes it unsuitable for most of the practical situations. In real-world situations, the variance-to-mean ratio frequently exceeds 1 (overdispersion), although in certain scenarios the ratio also falls below 1 (underdispersion) [10]. Consequently, researchers in recent times have shown keen interest in modelling these dispersion structures through flexible probability models and their regression counterparts.

The negative binomial distribution, a mixture of Poisson and gamma distributions, has served as an effective traditional alternative to handle overdispersion. Lawless [21] and Hilbe [18] have discussed this framework extensively. Similarly, Consul and Famoye [11]

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