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Research Article

A New Probability Model Based on a Coherent System with Applications

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The notion of a coherent system allows us to formalize how the random lifetime of the system is connected to the random lifetimes of its components. These connections are also generators of new pliant distributions, being those of various mixes of minimum and maximum of random variables. In this paper, a new four-parameter lifetime probability distribution is introduced by using the notion of a coherent system. Its structural properties are assessed and evaluated, including the analytical study of its main functions, stochastic dominance results, moments, and moment generating function. The proposed distribution, in particular, is proving to be efficient at fitting data with slight negative skewness and platykurtic as well as leptokurtic nature. This is illustrated by the analysis of three relevant real-life data sets, two in reliability and another in production, exhibiting the significance of the introduced model in comparison to various well-known models in statistical literature.

1. Introduction

Coherent systems are very important in reliability theory and data analysis. A n-component system is said to be coherent if its structure function is monotonic (that is, the improvement of components cannot lead to a deterioration in system performance), and it contains no irrelevant components (that is, all components have an effect on system performance). The detailed descriptions of various coherent systems can be found in [1–3]. Many authors have studied the reliability properties of coherent systems. Special attention has been paid to independent and identically distributed coherent systems and, in particular, to k out-of-n (order statistics), parallel, and series systems. Recently, some authors have started to study systems with dependence structures (see, for example, [4–7]). In this study, the following coherent system is considered. Consider a system having three components, numbered by I, II, and III, the component III playing a central role. Assume that the components I, III, and II are ordered in a straight line, and that the system works if there are at least two consecutively working components, i.e., IN $\mapsto (I) \mapsto (III) \rightleftharpoons (II) \mapsto$ OUT, where for instance, $(I) \rightleftharpoons (II)$ means that I and III are consecutive and connected; if one of the two component falls, their connection ends. This system is evoked in [8] as a consecutive 2-out-of-3 system. Hence, the lifetimes of the components I, II, and III can be modeled as three random variables, say U_1, U_2, and U_3, respectively. Here, we suppose that they are independent and subjected to the following distributional assumptions: U_1 follows the exponential distribution with parameter $\lambda_1 > 0$, U_2 follows the exponential distribution with parameter $\lambda_2 > 0$, and U_3 follows the Weibull