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Modelling of Large Elasto-Plastic Deformations By EFGM

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Abstract

Current work reports modelling and simulation of geometric and material nonlinearities arising due large elasto-plastic displacements in structural specimens by invoking enriched element free Galerkin method (EFGM). The displacement approximations are constructed by using moving least square approach. Standard displacement based approximations are modified by incorporating suitable enrichment functions depending on the nature of interfaces present in the components. Large deformations give rise to geometric nonlinearities which have been modelled by invoking total Lagrangian approach in which the initial unloaded state is chosen as the reference state for investigation. One of the main advantages of total Lagrangian approach lies in the selection of reference configuration which remains same throughout the simulation. Elastic-predictor-plastic-corrector algorithm has been used for the estimation of stresses during simulation. Mathematical foundations on EFGM are programmed in MATLAB to solve different engineering problems. Finally, various nonlinear problems are reported to establish the potential of enriched EFGM in modelling geometric and material nonlinearities in bi-material structural components. The results obtained in the current work are compared with finite element and coupled FE-EFG solutions so that the potential and accuracy of the proposed approach are established.

Keywords: EFGM, bi-material interfaces, elasto-plasticity, large deformation, enrichment functions

INTRODUCTION

In the past several decades, extensive studies and research have been conducted to investigate geometric and material nonlinearities that may occur in structural components having different types of internal interfaces in the domain. These discontinuities may arise due to cracks, holes or bi-material surfaces, as shown in Figure. 1. Several computational approaches have been proposed from time to time to simulate different engineering problems, which include boundary element methodology (BEM) [1-3], finite element method (FEM) [4], meshless techniques [5–8] and extended FEM (XFEM) [9,10]. Although FEM provides the most efficient and powerful computational technique for investigating different solid mechanics problems but it faces several problems while investigating different types of internal interfaces, because of the requirement of FEM mesh to conform to the orientation of the interfaces. Creating a conformal finite element mesh for irregularly shaped geometrical interfaces has always created problems in computational mechanics. The requirement of mesh conformation, mesh refinements and remeshing makes conventional finite element method more demanding and expensive as compared to XFEM. FEM also experiences extreme mesh distortion while modelling large deformation problems that involve geometric nonlinearities in the domain of interest.

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Geometrically nonlinear large deformation problems are handled more efficiently by meshless techniques such as EFGM. All mesh related issues are eliminated in EFGM which makes it a novel computational tool for modelling large or significant elasto-plastic deformations occurring in the domain.

XFEM was developed as the extended version of conventional FEM, which provided an efficient computational tool for modelling the material interfaces present in the structural components like holes, cracks or bi-material interfaces. This technique does not require conformal meshing to model these geometrical discontinuities. The discontinuities are investigated by enriching conventional EFGM approximations with suitable enrichment functions [11]. Various problems involving the modelling of discontinuities have been solved by XFEM, which include quasi-static crack growth in structural components [12], cohesive crack growth in engineering materials [13-15], cyclic crack growth [16-18], propagating and non-propagating cracks [19] and 3D crack growth [20,21], crack nucleation and multi-crack cases [22–25], holes and inclusions [26, 27], problems involving bi-material discontinuities [28–33], frictional contact between solid bodies [34–37], time dependent crack growth [38-42] and nonlinear crack growth cases [43–45]. This method was also successfully invoked to model several problems in the areas of fluid dynamics [46-51], problems involving phase changes [52–54], piezoelectric cases [55] and other complex mechanical components [56,57].

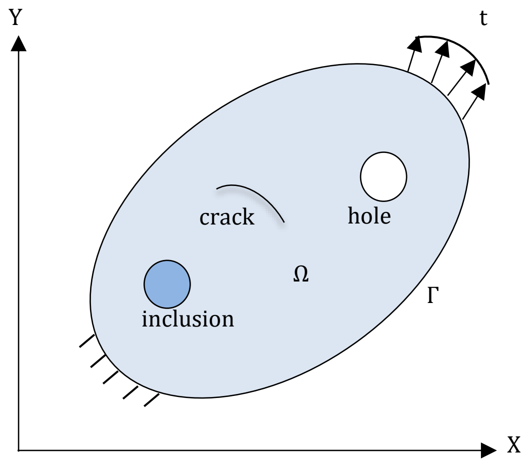

Figure 1. Domain with different types of discontinuities

The current work provides an efficient and accurate numerical technique, based on enriched EFGM, to investigate geometrical and material nonlinearities in bi-material structural components. Several specimens containing internal interfaces have been discussed in the current work. Large deformations have been modelled by the total Lagrangian approach which considers initial unloaded state as the reference state for investigation. Deformation gradient tensor has been used to map the data between different configurations during large deformation. Mathematical foundations on EFGM are programmed in MATLAB to investigate several engineering problems. Finally, various numerical problems are reported to establish the potential of enriched EFGM in modelling geometrical and material nonlinearities in bi-material structural components.

FUNDAMENTALS OF ENRICHED EFGM

EFGM was proposed with the goal of eliminating the limitations related to the dependence of FE mesh to construct the approximate solution. In all meshless techniques, the structural component is represented by an array of nodes and displacement approximation is derived from the knowledge of these nodes only. Generation of finite element mesh is not required in meshless techniques. Smooth particle hydrodynamics (SPH) was among the first meshfree methods that was developed to model various astrophysical phenomena including dust clouds and exploding stars. This technique was later

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applied to investigate different solid mechanics problems. Conventional smooth particle hydrodynamics method suffered various limitations while modelling solid mechanics problems like instabilities, inconsistencies, due to which modifications were carried out so that this technique becomes suitable for computational solid mechanics. SPH technique and its updated versions were based on strong form of mathematical model. On the contrary, EFGM was developed considering the weak form of original mathematical models. All the mesh related problems are eliminated in EFGM, which proves to be very useful for solving large deformation problems where mesh distortions are predominant, and the conventional finite element method suffers serious drawbacks in those cases. While modelling such problems by the FEM, severe mesh distortions occur which makes further analysis difficult and remeshing is required. However, these problems do not arise in meshless techniques. The performance of all meshless techniques is seriously influenced by selection of weight functions, which have a compact support that defines the domain of influence of the concerned nodes.

Approximation Function in EFGM

As already explained, EFGM constructs the displacement approximations with the help of nodal distributions only. Moving least square (MLS) based displacement approximants are used for describing the displacements. Such approximations include a polynomial basis, weight function and coefficients dependent on position of node. The MLS approximation for primary variable $u^h(x)$ can be written as

$$u^h(x) = \sum_{j=1}^{n} \mathrm{p}_j(x)\mathrm{a}_j(x) = \mathrm{P}^T(x)\mathrm{a}(x) \tag{1}$$

where, n denotes nodes in neighbourhood of point x where weight function is not equal to zero, $\mathrm{p}_j(x)$ represents polynomial basis, $\mathrm{a}_j(x)$ are coefficients dependent on the position of node. The polynomial basis functions for different cases can be defined as

$$\mathrm{P}^T(x) = [1, x] \quad linear - one\ dimensional \tag{2}$$
$$\mathrm{P}^T(x) = [1, x, y] \quad linear - two\ dimensional \tag{3}$$
$$\mathrm{P}^T(x) = [1, x, y, x^2, y^2, xy] \quad quadratic - two\ dimensional \tag{4}$$
$$\mathrm{P}^T(x) = [1, x, y, x^2, y^2, xy, x^3, y^3, x^2y, xy^2] \quad cubic - two\ dimensional \tag{5}$$

Coefficients $\mathrm{a}(x)$ are derived by minimizing the weighted least square sum $L(x)$ of the error between u^h and nodal parameter u_i, which is written as

$$L(x) = \sum_{j=1}^{n} w_j(x)[\mathrm{P}^T(x)\mathrm{a}(x) - u_i]^2 \tag{6}$$

$L(x)$ is minimized as

$$\frac{\partial L(x)}{\partial \mathrm{a}(x)} = 0 \tag{7}$$

After carrying out the above differentiation, we reach at

$$\mathrm{A}(x)\mathrm{a}(x) = \mathrm{B}(x)\mathrm{u} \tag{8}$$

Where

$$\mathrm{u}^T = [u_1,\ u_2, u_3, \dots\dots\dots\dots\dots\dots.\, u_n] \tag{9}$$
$$\mathrm{A}(x) = \sum_{j=1}^{n} w_j(x)\mathrm{P}(x_j)\mathrm{P}^T(x_j) \tag{10}$$
$$\mathrm{B}(x) = [w_1(x)\mathrm{P}(x_1),\ w_2(x)\mathrm{P}(x_2), \dots\dots\dots, w_n(x)\mathrm{P}(x_n)] \tag{11}$$

Coefficients $\mathrm{a}(x)$ are derived as $\mathrm{a}(x) = \mathrm{A}^{-1}(x)\mathrm{B}(x)\mathrm{u}$. Now, $u(x)$ is written as

$$u^h(x) = \mathrm{P}^T(x)\mathrm{A}^{-1}(x)\mathrm{B}(x)\mathrm{u} \tag{12}$$

which further gives

$$u^h(x) = \sum_{i=1}^{n} \mathrm{N}_i(x)u_i = \mathrm{N}^T(x)u \quad (13)$$

where N_i are the MLS shape functions described as

$$\Psi_i = \mathrm{P}^T(x)\mathrm{A}^{-1}(x)\mathrm{B}(x) \quad (14)$$

The derivatives of MLS shape functions are written as

$$\mathrm{N}_{i,x} = \left(\mathrm{P}^T(x)\mathrm{A}^{-1}(x)\mathrm{B}(x)\right)_{,x} = (\mathrm{P}^T)_{,x}\mathrm{A}^{-1}\mathrm{B} + \mathrm{P}^T(\mathrm{A}^{-1})_{,x}\mathrm{B} + \mathrm{P}^T\mathrm{A}^{-1}\mathrm{B}_{,x} \quad (15)$$

where

$$\mathrm{B}_{,x} = w_{i,x}\mathrm{P}(x_i) \quad (16)$$

$$(\mathrm{A}^{-1})_{,x} = -\mathrm{A}^{-1}\mathrm{A}_{,x}\mathrm{A}^{-1} \quad (17)$$

$$\mathrm{A}_{,x} = \sum_{j=1}^{n} w_{i,x}\,\mathrm{P}(x_j)\mathrm{P}^T(x_j) \quad (18)$$

Choice of Weight Function

Proper selection of weight functions is the most significant feature of meshfree techniques. The weight functions should have a positive value inside their domain and must decrease monotically with increasing distance from the node. Several categories of weight functions have been proposed for EFGM, out of which the important ones include the exponential, cubic and quartic spline form of weight functions. The most commonly used form of weight function in EFGM is the quartic spline. Some of the weight functions are given below

(1) Cubic spline

$$w(r) = \begin{cases} \frac{2}{3} - 4r^2 + 4r^3, & r \le \frac{1}{2} \\ \frac{4}{3} - 4r + 4r^2 - \frac{4}{3}r^3, & \frac{1}{2} < r \le 1 \\ 0, & r > 1 \end{cases} \quad (19)$$

(2) Quartic spline

$$w(r) = \begin{cases} 1 - 6r^2 + 8r^3 - 3r^4, & r \le 1 \\ 0, & r > 1 \end{cases} \quad (20)$$

where

$$r = \frac{\|x_I - x\|}{d_I} \quad (21)$$

Here, x_I represents the node and d_I is its domain of influence. In two dimensional problems, circular and rectangular supports have found extensive applications. The size of compact support plays a crucial role in the behaviour of meshfree methods. Dolbow and Belytschko defined the size of compact support as $d_I = d_{max}c_I$, where c_I is chosen in such a way that good number of nodes are available in the support and the value of d_{max} lies between 2.0 and 4.0.

MODELLING OF BI-MATERIAL INTERFACES BY EFGM

The bi-material discontinuity produces a displacement field that remains continuous across the interface, but it produces a discontinuity in strain field. Such discontinuities are investigated by choosing two enrichment functions. One of the enrichment functions is described as

$$\mathrm{F(x)} = \left|\sum_{\mathrm{i}} \phi_{\mathrm{i}}\mathrm{N_i(x)}\right| \quad (22)$$

where ϕ represents the level set function. Another function is expressed as

$$\mathrm{F(x)} = \sum_{\mathrm{i}} |\phi_{\mathrm{i}}|\mathrm{N_i(x)} - \left|\sum_{\mathrm{i}} \phi_{\mathrm{i}}\mathrm{N_i(x)}\right| \quad (23)$$

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The modified EFGM approximation is written as

$$u^h(x) = \sum_{i=1}^{n} N_i(x)u_i + \sum_{i=1}^{n_s} N_i(x)\,[F(x) - F(x_i)]a_i \tag{24}$$

The final EFGM model is derived after the substitution of variable approximation in the equilibrium equation, which gives $[K^e]\{d^e\} = \{f^e\}$, where

$$[K^e] = \begin{bmatrix} K^{uu} & K^{ua} \\ K^{au} & K^{aa} \end{bmatrix} \;\; ; \;\; \{f^e\} = \{f^u \;\; f^a\}^T \;\; ; \;\; \{d^e\} = \{u \;\; a\}^T \tag{25}$$

$$K^{rs} = \int_{\Omega^e} (B^r)^T D B^s d\Omega \qquad \text{where } r, s = u, a \tag{26}$$

$$f^u = \int_{\Omega^e} N^T b \, d\Omega + \int_{\Gamma^e} N^T t \, d\Gamma \tag{27}$$

$$f^a = \int_{\Omega^e} N^T \big(F(x) - F(x_i)\big) b \, d\Omega + \int_{\Gamma^e} N^T (F(x) - F(x_i)) t \, d\Gamma \tag{28}$$

$$B^u = \begin{bmatrix} N_{i,x} & 0 \\ 0 & N_{i,y} \\ N_{i,y} & N_{i,x} \end{bmatrix} \;\; ; \;\; B^a = \begin{bmatrix} (N_i(F(x) - F(x_i)))_{,x} & 0 \\ 0 & (N_i(F(x) - F(x_i)))_{,y} \\ (N_i(F(x) - F(x_i)))_{,y} & (N_i(F(x) - F(x_i)))_{,x} \end{bmatrix} \tag{29}$$

Modelling of Large Deformation by EFGM

The current work employs the total Lagrangian approach to investigate the geometric nonlinearity caused by large deformation. The equilibrium equation for modelling geometric nonlinearities is written as

$$\int_{\Omega} \delta F^T \, P d\Omega - \int_{\Omega} \delta u^T b \, d\Omega - \int_{\Gamma_t} \delta u^T t \, d\Gamma_t = 0 \tag{30}$$

where F denotes deformation gradient tensor defined as $F_{ij} = \frac{\partial x_i}{\partial X_j} = \frac{\partial u_i}{\partial X_j} + \delta_{ij}$, b denotes body force vector, P represents first Piola-Kirchoff stresses, t is applied traction vector, $x = [x \quad y]^T$ denotes deformed configuration of the given body and $X = [X \quad Y]^T$ denotes the unloaded state of the body. We generally use second Piola-Kirchoff stress S, which can be obtained as $P = SF^T$.

Substitution of EFGM based displacement approximation in the equilibrium equation yields the EFGM model as

$$\begin{bmatrix} K^{uu} & K^{ua} \\ K^{au} & K^{aa} \end{bmatrix} \begin{Bmatrix} u \\ a \end{Bmatrix} = \begin{Bmatrix} f^u \\ f^a \end{Bmatrix} \tag{31}$$

where

$$K^{\alpha\beta} = \int_{\Omega^e} (B^\alpha)^T D^{ep} \, B^\beta d\Omega + \int_{\Omega^e} (G^\alpha)^T M_s \, G^\beta d\Omega \qquad (\alpha, \beta = u, a) \tag{32}$$

$$f^u = \int_{\Omega^e} N^T \bar{b} \, d\Omega + \int_{\Gamma^e} N^T \bar{t} d\Gamma \tag{33}$$

$$f^a = \int_{\Omega^e} N^T \big(F(X) - F(X_i)\big) \bar{b} \, d\Omega + \int_{\Gamma^e} N^T (F(X) - F(X_i)) \bar{t} \, d\Gamma \tag{34}$$

$$M_s = \begin{bmatrix} S_{XX} & 0 & S_{XY} & 0 \\ 0 & S_{XX} & 0 & S_{XY} \\ S_{XY} & 0 & S_{YY} & 0 \\ 0 & S_{XY} & 0 & S_{YY} \end{bmatrix} \tag{35}$$

$$B^u = \begin{bmatrix} N_{i,X} & 0 \\ 0 & N_{i,Y} \\ N_{i,Y} & N_{i,X} \end{bmatrix} \;\; ; \;\; B^a = \begin{bmatrix} (N_i(F(X) - F(X_i)))_{,X} & 0 \\ 0 & (N_i(F(X) - F(X_i)))_{,Y} \\ (N_i(F(X) - F(X_i)))_{,Y} & (N_i(F(X) - F(X_i)))_{,X} \end{bmatrix} \tag{36}$$

$$G^u = \begin{bmatrix} N_{i,X} & 0 \\ 0 & N_{i,X} \\ N_{i,Y} & 0 \\ 0 & N_{i,Y} \end{bmatrix} \;\; ; \;\; G^a = \begin{bmatrix} (N_i(F(X) - F(X_i)))_{,X} & 0 \\ 0 & (N_i(F(X) - F(X_i)))_{,X} \\ (N_i(F(X) - F(X_i)))_{,Y} & 0 \\ 0 & (N_i(F(X) - F(X_i)))_{,Y} \end{bmatrix} \tag{37}$$

In the above equations, D^{ep} is the elasto-plastic constitutive matrix, $\bar{b}$ and $\bar{t}$ are defined as $\bar{b} = \frac{b}{J}$ and $\bar{t} = t(\frac{da}{dA_0})$. J represents the determinant of the deformation gradient matrix, da denotes the deformed elemental area and dA_0 represents the original undeformed area. b and t represent body forces and surface tractions applied on the deformed configuration, respectively.

NUMERICAL RESULTS AND DISCUSSIONS

Few numerical problems are reported here to illustrate the applicability of EFGM in solving large-plasticity deformations in structural components. The first example reports the die pressing phenomenon in a bi-material specimen with horizontal interface between two materials. The second example investigates the die pressing phenomenon in a bi-material specimen with two horizontal interfaces, separating the harder material and the soft portion. These two cases are investigated by enriched EFGM and the results obtained during simulation are compared with FEM and coupled solutions, which are considered as the reference solution for validation. In both problems, the Young's modulus of weak portion is assumed as 2.1×10^8 N/m^2 with a Poisson ratio of 0.35. The von-Mises yield criterion has been used and yield stress of 2.4×10^5 N/m^2 has been assumed. The hardening parameter of 3×10^7 N/m^2 is considered for analysis. The stronger portion is elastic and has the Young's modulus of 2.1×10^9 N/m^2 with the Poisson ratio of 0.35.

Die Pressing of a Bi-Material Rectangular Object

Large deformation has been investigated in a rectangular component with one internal bi-material interface, as depicted in Figure 2. The given specimen is fixed at the top surface, and the lower edge is compressed to have 12 mm height reduction. The EFGM mesh of 26×41 nodes is chosen for the given domain, as can be seen in Figure. 3. Final deformed state for the given compaction is presented in Figure 4. Figure 5 shows the distribution of normal stress σ_{yy} along the common interface. The normal stress σ_{yy} along the vertical left edge can be seen in Figure. 6. Results derived in the current work show a close agreement with conventional FEM and coupled solutions available in literature and hence it is established that the presented approach can be effectively and efficiently used to study and investigate the problems involving geometric and material nonlinearities.

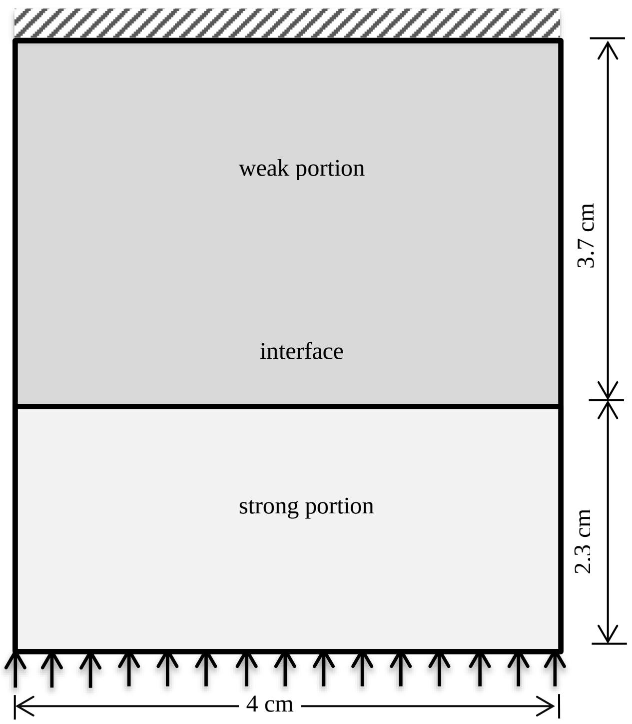

Figure 2. Rectangular component containing a horizontal interface

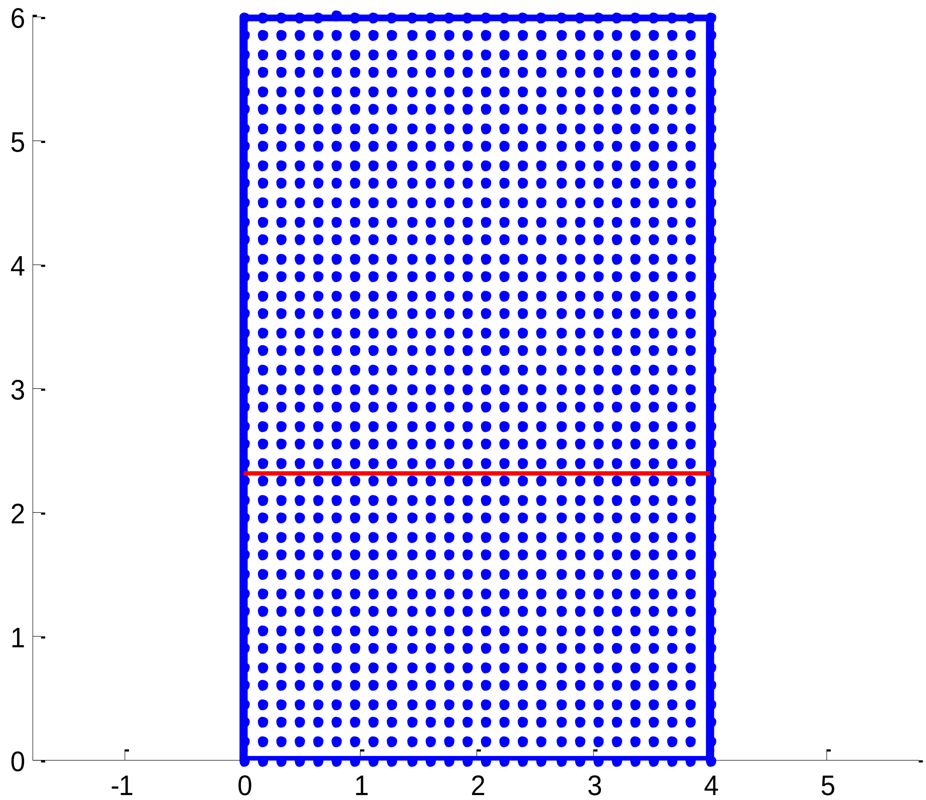

Figure 3. EFGM domain representation for one horizontal interface inside the domain

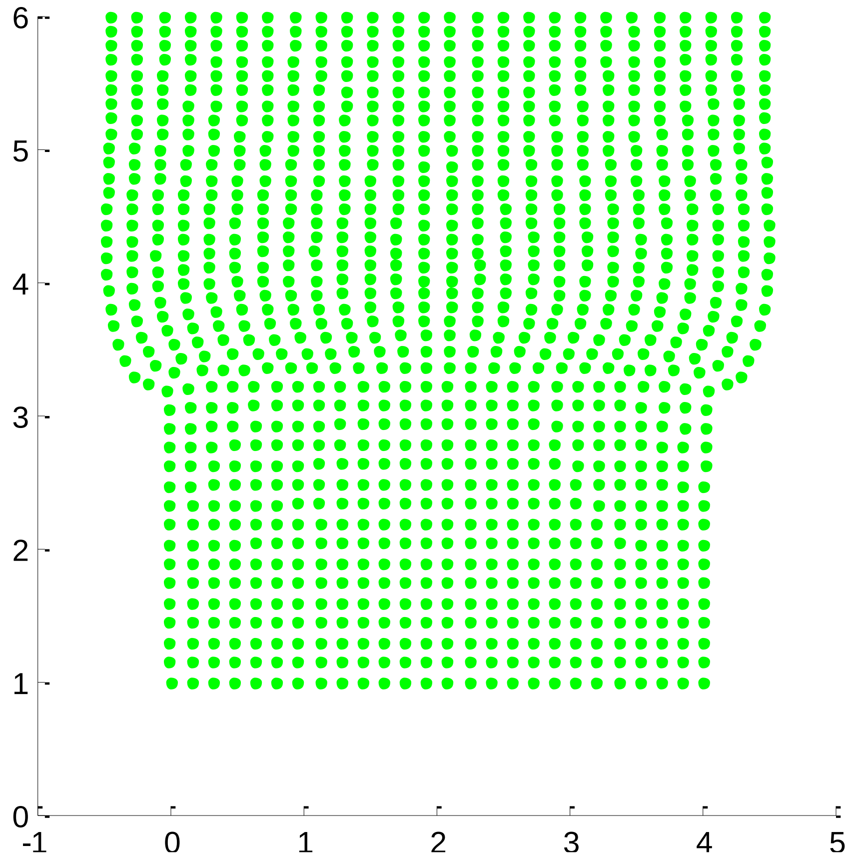

Figure 4. Deformed configuration obtained by EFGM.

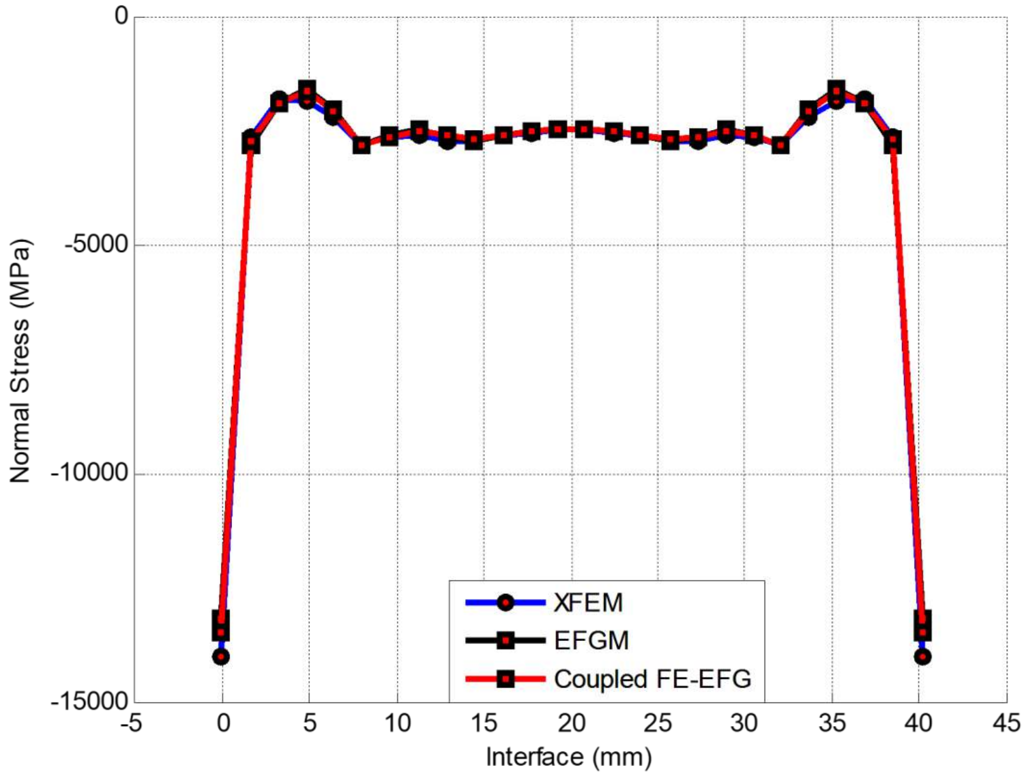

Figure 5. Normal stress σ_{yy} along the common interface

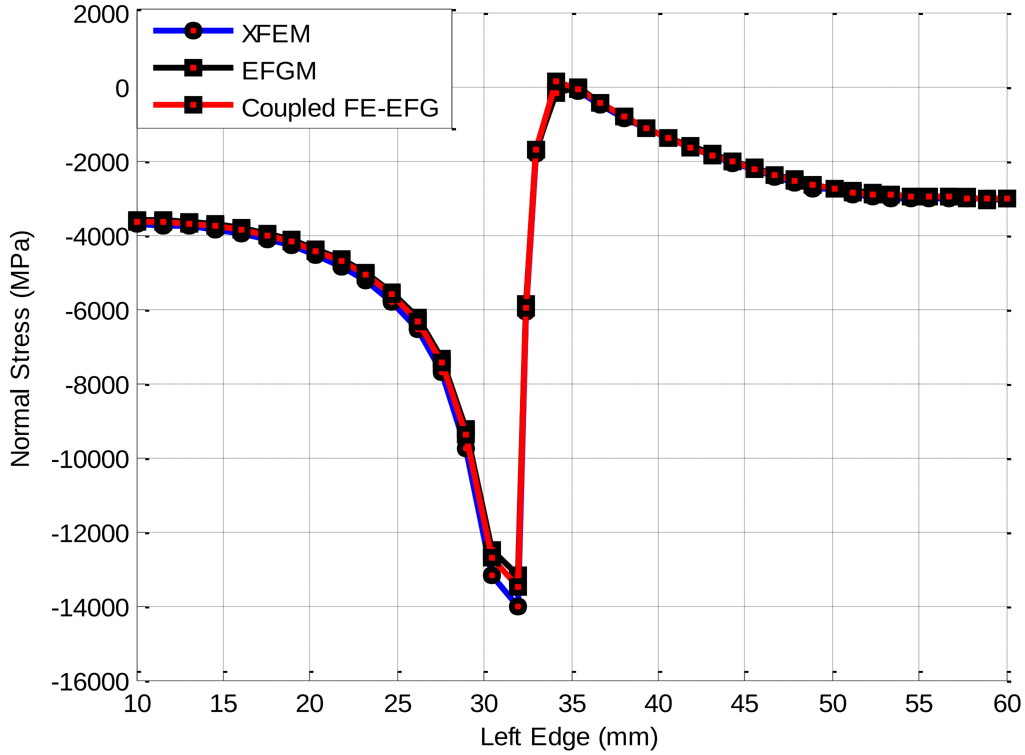

Figure 6. Normal stress σ_{yy} along the left vertical edge

Die pressing of a rectangular component between two hard dies

This example reports large deformation in a rectangular component with two internal bi-material interfaces, as can be seen in Figure. 7. Top edge of the component is restrained while a compaction is applied at the lower edge equal to 7.5 mm height reduction. Nodal array of 26×41 nodes is selected for the analysis, as depicted in Figure 8. Deformed configuration for the given compaction is presented in Figure 9. Figure. 10 shows the normal stress σ_{yy} along the common interface. Figure 11 presents the normal stress σ_{yy} across top edge of the component. Results derived in the current work show a close agreement with conventional FEM and coupled solutions available in literature and hence it is established that the presented approach can be effectively and efficiently used to study and investigate the problems involving geometric and material nonlinearities.

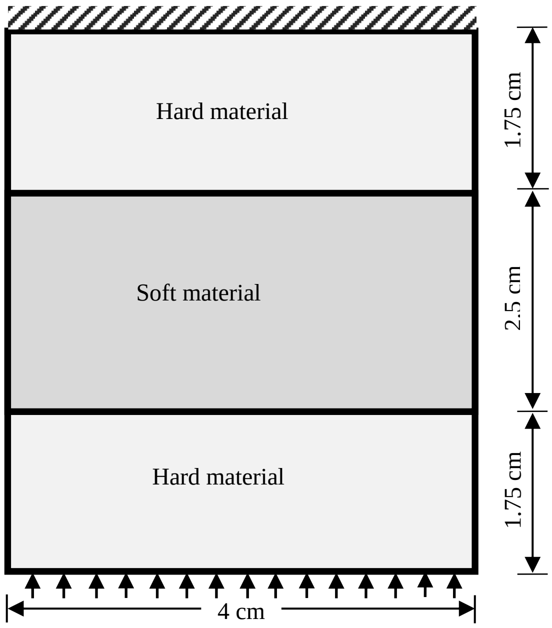

Figure 7. Rectangular component with two horizontal interfaces in the domain

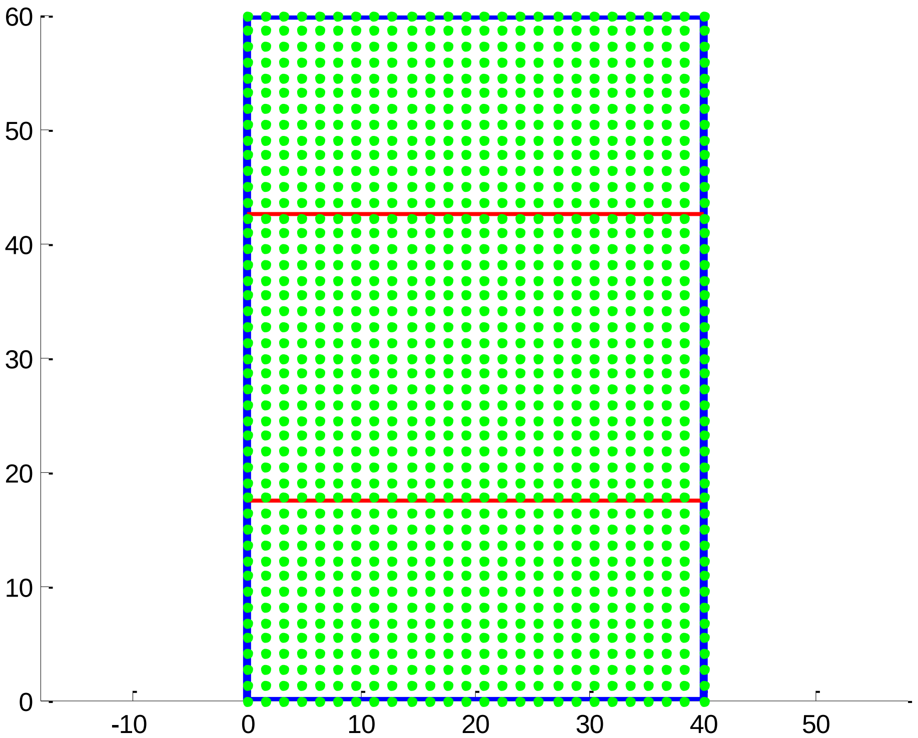

Figure 8. EFGM domain representation for a rectangular component with two horizontal interfaces

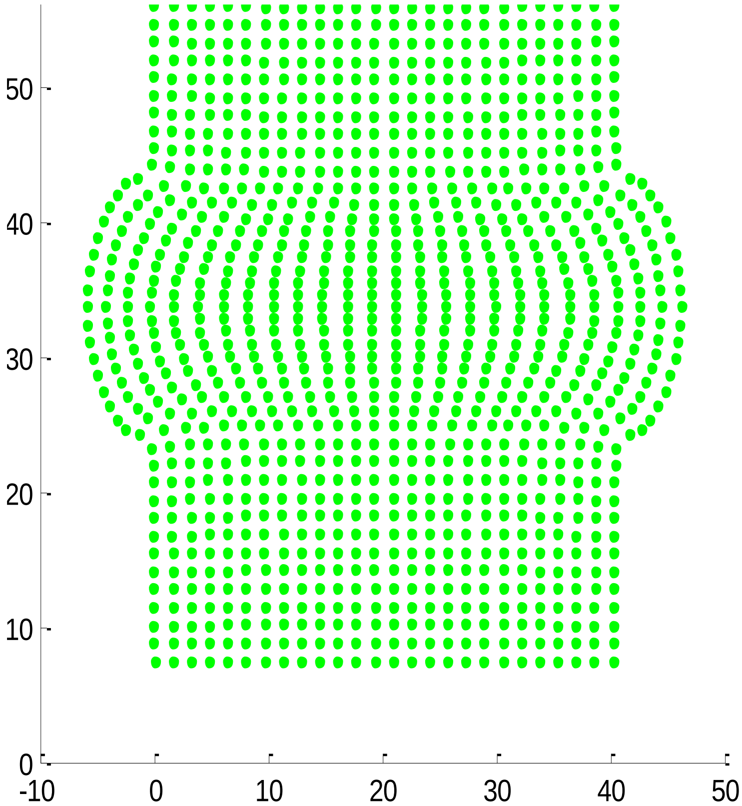

Figure 9. Deformed configuration obtained by EFGM.

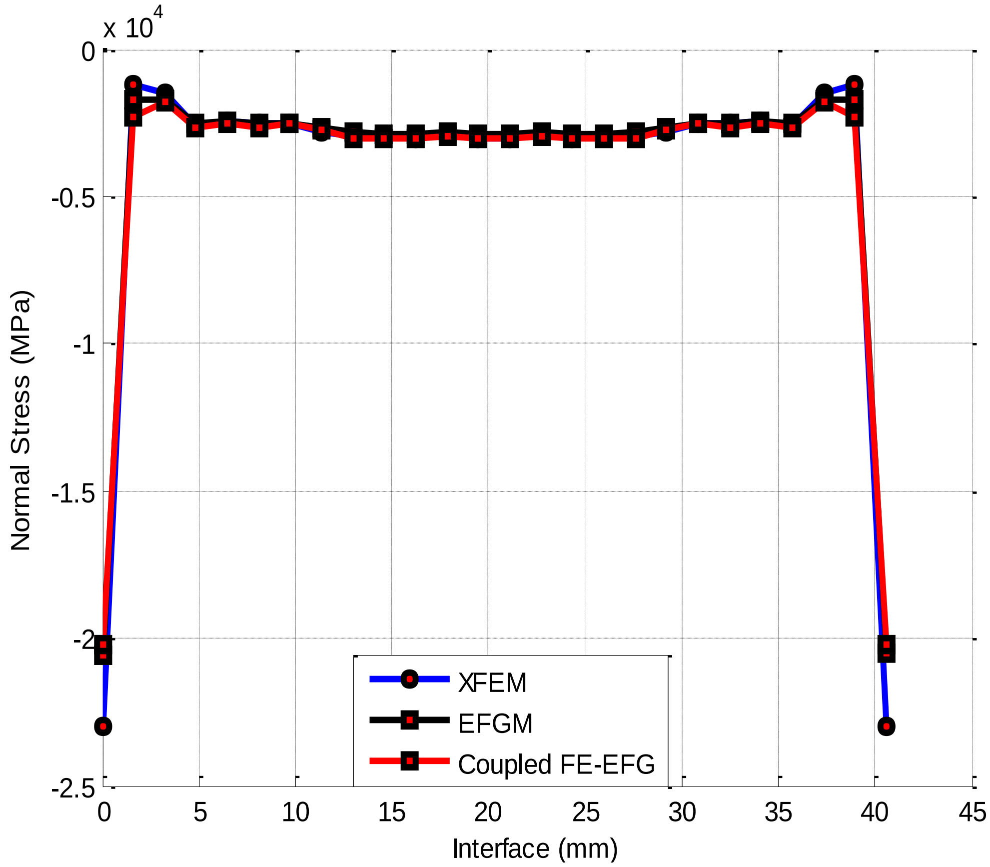

Figure 10. Normal stress σ_{yy} along the common interface

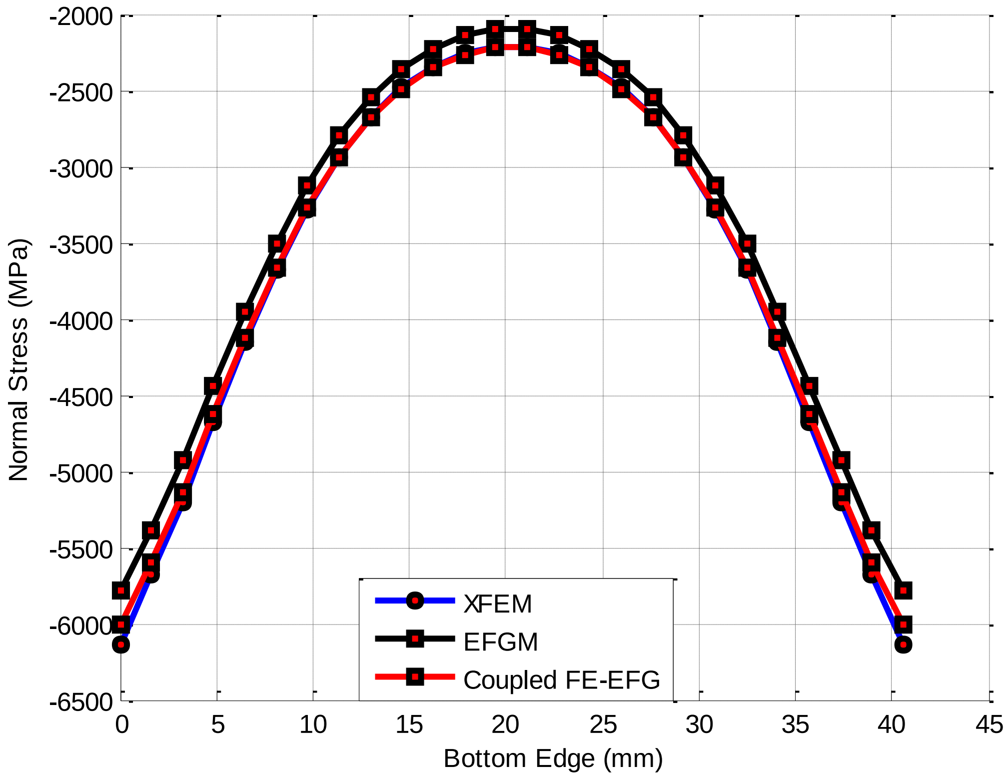

Figure 11. Normal stress σ_{yy} along the top surface

CONCLUSIONS

In the current work, enriched EFGM is invoked to model geometric and material nonlinearities in structural components containing internal interfaces. The conventional moving least square approximation is modified with suitable enrichment functions to model the influence created by various types of internal interfaces present in structural components. Results derived in the current work show a close agreement with conventional FEM and coupled solutions available in literature and hence it is established that the presented approach can be effectively and efficiently used to study and investigate the problems involving geometric and material nonlinearities.

REFERENCES

1. Portela A, Aliabadi MH, Rooke DP. The dual boundary element method: Effective implementation for crack problems. International Journal for Numerical Methods in Engineering. 1992;33(6):1269–87.
2. Yan AM, Nguyen-Dang H. Multiple-cracked fatigue crack growth by BEM. Computational Mechanics. 1995;16(5):273–80.
3. Xiangqiao Yan. A boundary element modeling of fatigue crack growth in a plane elastic plate. Mechanics Research Communications. 2006; 33:470–81.
4. Cheung S, A.R. Luxmoore. A finite element analysis of stable crack growth in an aluminium alloy. Engineering Fracture Mechanics. 2003;70(9):1153–69.
5. Belytschko T, Lu YY, Gu L. Crack propagation by element-free Galerkin methods. Engineering Fracture Mechanics. 1995;51(2):295–315.
6. Jha A, Sarkar S, Singh IV, Mishra BK, Singh R, Singh RN. A study on the effect of residual stresses on hydride assisted crack in Zr-2.5Nb pressure tube material using XFEM. Theoretical and Applied Fracture Mechanics. 2022 Oct; 121:103536.
7. Duflot M, Nguyen-Dang H. A meshless method with enriched weight functions for fatigue crack growth. International Journal for Numerical Methods in Engineering. 2004;59(14):1945–61.
8. Duflot M, Nguyen-Dang H. Fatigue crack growth analysis by an enriched meshless method. Journal of Computational and Applied Mathematics. 2004;168(1-2):155–64.

9. Belytschko T, Black T. Elastic crack growth in finite elements with minimal remeshing. International Journal for Numerical Methods in Engineering. 1999;45(5):601–20.
10. Daux C, Moes N, Dolbow J, Sukumar N, Belytschko T. Arbitrary branched and intersecting cracks with the extended finite element method. International Journal for Numerical Methods in Engineering. 2000;48(12):1741–60.
11. Jena J, Singh IV, Gaur V. XFEM for semipermeable crack in piezoelectric material with Maxwell stress. Engineering Fracture Mechanics. 2023 Jun 1; 285:109281–1.
12. Sukumar N, Prevost JH. Modeling quasi-static crack growth with the extended finite element method Part I: Computer implementation. International Journal of Solids and Structures. 2003;40(26):7513–37.
13. Zi G, Belytschko T. New crack-tip elements for XFEM and applications to cohesive cracks. International Journal for Numerical Methods in Engineering. 2003;57(15):2221–40.
14. Unger JF, Eckardt S, Könke C. Modelling of cohesive crack growth in concrete structures with the extended finite element method. Computer Methods in Applied Mechanics and Engineering. 2007;196(41-44):4087–100.
15. Asferg JL, Peter Noe Poulsen, Leif Otto Nielsen. A consistent partly cracked XFEM element for cohesive crack growth. International Journal for Numerical Methods in Engineering. 2007;72(4):464–85.
16. Qian J, Fatemi A. Mixed mode fatigue crack growth: A literature survey. Engineering Fracture Mechanics. 1996;55(6):969–90.
17. Chopp DL, Sukumar N. Fatigue crack propagation of multiple coplanar cracks with the coupled extended finite element/fast marching method. 2003;41(8):845–69.
18. Stolarska M, Chopp DL. Modeling thermal fatigue cracking in integrated circuits by level sets and the extended finite element method. International Journal of Engineering Science. 2003;41(20):2381–410.
19. Ventura G, Élisa Budyn, Belytschko T. Vector level sets for description of propagating cracks in finite elements. International Journal for Numerical Methods in Engineering. 2003;58(10):1571–92.
20. Areias PMA, Belytschko T. Analysis of three-dimensional crack initiation and propagation using the extended finite element method. International Journal for Numerical Methods in Engineering. 2005;63(5):760–88.
21. Sukumar N, Moes N, Moran B, Belytschko T. Extended finite element method for three-dimensional crack modelling. International Journal for Numerical Methods in Engineering. 2000;48(11):1549–70.
22. Giner E, Sukumar N, Denia FD, Fuenmayor FJ. Extended finite element method for fretting fatigue crack propagation. 2008;45(22-23):5675–87.
23. Budyn É, Zi G, Moës N, Belytschko T. A method for multiple crack growth in brittle materials without remeshing. International Journal for Numerical Methods in Engineering. 2004;61(10):1741–70.
24. Jameel A, Harmain GA. Modeling and Numerical Simulation of Fatigue Crack Growth in Cracked Specimens Containing Material Discontinuities. Strength of Materials. 2016;48(2):294–307.
25. Bellec J, Dolbow JE. A note on enrichment functions for modelling crack nucleation. Communications in Numerical Methods in Engineering. 2003;19(12):921–32.
26. Kanth SA, Harmain GA, Jameel A. Modeling of Nonlinear Crack Growth in Steel and Aluminum Alloys by the Element Free Galerkin Method. Materials Today: Proceedings. 2018;5(9):18805–14.
27. Duhan N, Mishra BK, Singh IV. XFEM for multiphysics analysis of edge dislocations with nonuniform misfit strain: A novel enrichment implementation. Computer Methods in Applied Mechanics and Engineering. 2023; 413:116079–9.
28. Belytschko T, Parimi C, Moës N, Sukumar N, Usui S. Structured extended finite element methods for solids defined by implicit surfaces. International Journal for Numerical Methods in Engineering. 2002;56(4):609–35.
29. Singh AK, Jameel A, Harmain GA. Investigations on crack tip plastic zones by the extended iso-geometric analysis. Materials Today: Proceedings. 2018;5(9):19284–93.

30. Hettich T, Ramm E. Interface material failure modeled by the extended finite-element method and level sets. Computer Methods in Applied Mechanics and Engineering. 2006;195(37-40):4753–67.
31. Asadpoure A, Mohammadi S, Vafai A. Crack analysis in orthotropic media using the extended finite element method. Thin-Walled Structures. 2006;44(9):1031–8.
32. Kanth SA, Lone AS, Harmain GA, Jameel A. Elasto Plastic Crack Growth by XFEM: A Review. Materials Today: Proceedings. 2019; 18:3472–81.
33. Jameel A, Harmain GA. Fatigue crack growth analysis of cracked specimens by the coupled finite element-element free Galerkin method. Mechanics of Advanced Materials and Structures. 2018;26(16):1343–56.
34. Vitali E, Benson DJ. An extended finite element formulation for contact in multi-material arbitrary Lagrangian–Eulerian calculations. International Journal for Numerical Methods in Engineering. 2006;67(10):1420–44.
35. Jameel A, Harmain GA. Extended iso-geometric analysis for modeling three-dimensional cracks. Mechanics of Advanced Materials and Structures. 2018;26(11):915–23.
36. Lone AS, Kanth SA, Jameel A, Harmain GA. A state of art review on the modeling of Contact type Nonlinearities by Extended Finite Element method. Materials Today: Proceedings. 2019; 18:3462–71.
37. Lone AS, Jameel A, G.A. Harmain. A coupled finite element-element free Galerkin approach for modeling frictional contact in engineering components. Materials Today: Proceedings. 2018;5(9):18745–54.
38. Jameel A, G.A. Harmain. A coupled FE-IGA technique for modeling fatigue crack growth in engineering materials. 2019;26(21):1764–75.
39. Belytschko T, Chen H, Xu J, Zi G. Dynamic crack propagation based on loss of hyperbolicity and a new discontinuous enrichment. International Journal for Numerical Methods in Engineering. 2003;58(12):1873–905.
40. Julien Réthoré, Gravouil A, Alain Combescure. An energy-conserving scheme for dynamic crack growth using the eXtended finite element method. International Journal for Numerical Methods in Engineering. 2005;63(5):631–59.
41. Sheikh UA, Jameel A. Elasto-plastic large deformation analysis of bi-material components by FEM. Materials Today: Proceedings. 2020; 26:1795–802.
42. Rozycki P, Moes N, Bechet E, Dubois C. X-FEM explicit dynamics for constant strain elements to alleviate mesh constraints on internal or external boundaries. Computer Methods in Applied Mechanics and Engineering. 2008;197(5):349–63.
43. Kanth SA, Lone AS, Harmain GA, Jameel A. Modeling of embedded and edge cracks in steel alloys by XFEM. Materials Today: Proceedings. 2020; 26:814–8.
44. Hutchinson JW. Singular behaviour at the end of a tensile crack in a hardening material. Journal of The Mechanics and Physics of Solids. 1968;16(1):13–31.
45. Rice JR, Rosengren GF. Plane strain deformation near a crack tip in a power-law hardening material. Journal of The Mechanics and Physics of Solids. 1967;16(1):1–12.
46. Wagner GJ, Moës N, Liu WK, Belytschko T. The extended finite element method for rigid particles in Stokes flow. International Journal for Numerical Methods in Engineering. 2001;51(3):293–313.
47. Chessa J, Belytschko T. An Extended Finite Element Method for Two-Phase Fluids. Journal of Applied Mechanics. 2003;70(1):10–7.
48. Lone AS, Kanth SA, Harmain GA, Jameel A. XFEM modeling of frictional contact between elliptical inclusions and solid bodies. Materials Today: Proceedings. 2020; 26:819–24.
49. Sven Groí, Reusken A. An extended pressure finite element space for two-phase incompressible flows with surface tension. 2007;224(1):40–58.
50. Jameel A, Harmain GA. Effect of material irregularities on fatigue crack growth by enriched techniques. International Journal for Computational Methods in Engineering Science and Mechanics. 2020;21(3):109–33.
51. Sharma D, Singh IV, Kumar J. A computational framework based on 3D microstructure modelling to predict the mechanical behaviour of polycrystalline materials. International Journal of Mechanical Sciences. 2023; 258:108565–5.

52. Kanth SA, Harmain GA, Jameel A. Investigation of fatigue crack growth in engineering components containing different types of material irregularities by XFEM. Mechanics of Advanced Materials and Structures. 2021;1–39.
53. Ji H, Chopp DL, Dolbow JE. A hybrid extended finite element/level set method for modeling phase transformations. International Journal for Numerical Methods in Engineering. 2002;54(8):1209–33.
54. Jameel A, Harmain GA. Large deformation in bi-material components by XIGA and coupled FE-IGA techniques. Mechanics of Advanced Materials and Structures. 2020;1–23.
55. Éric Béchet, Scherzer M, Kuna M. Application of the X-FEM to the fracture of piezoelectric materials. International Journal for Numerical Methods in Engineering. 2008;77(11):1535–65.
56. Bordas S, Moran B. Enriched finite elements and level sets for damage tolerance assessment of complex structures. Engineering Fracture Mechanics. 2006;73(9):1176–201.
57. Lone AS, Harmain GA, Jameel A. Modeling of contact interfaces by penalty based enriched finite element method. Mechanics of Advanced Materials and Structures. 2022;30(7):1485–503.

