Received: 5 April 2023 | Revised: 20 July 2023 | Accepted: 25 September 2023

DOI: 10.1002/mma.9723

RESEARCH ARTICLE

WILEY

N-dimensional wave packet transform and associated uncertainty principles in the free metaplectic transform domain

Aamir Hamid Dar | **Mohammad Younus Bhat**

Department of Mathematical Sciences, Islamic University of Science and Technology, Kashmir, India

Correspondence
Mohammad Younus Bhat, Department of Mathematical Sciences, Islamic University of Science and Technology, Kashmir 192122, India.
Email: gyounusg@gmail.com

Communicated by: Q. Chen

Funding information
JKSTIC Research Project, Grant/Award Number: JKST&IC/SRE/J/357-60

The free metaplectic transformation (FMT) is an n-dimensional linear canonical transform. This transform is much useful, especially in multidimensional signal processing and applications. In this paper, our aim is to achieve an efficient time-frequency representation of higher-dimensional nonstationary signals by introducing the novel free metaplectic wave packet transform (FM-WPT) in $L^2(\mathbb{R}^n)$, based on the elegant convolution structure associated with the free metaplectic transforms. The FM-WPT preserves the properties of classical wave packet transform (WPT) in $L^2(\mathbb{R}^n)$ and has better mathematical properties. Further, the validity of the proposed transform is demonstrated via a lucid example. The preliminary analysis encompasses the derivation of fundamental properties of the novel FM-WPT, including boundedness, reconstruction formula, Moyal's formula, and the reproducing kernel. To extend the scope of the study, we formulate several uncertainty inequalities, including Lieb's inequality, Pitt's inequality, logarithmic inequality, Heisenberg's uncertainty inequality, and Nazarov's uncertainty inequality for the proposed transform.

KEYWORDS

convolution, free metaplectic transform, free metaplectic wave packet transform, free metaplectic wavelet, Heisenberg's UP, logarithmic's UP, Moyal's formula, Nazarov's UP, symplectic matrix, uncertainty principle(UP)

MSC CLASSIFICATION

47b38, 42C40, 42B10, 70h15

1 | INTRODUCTION

The n-dimensional linear canonical transformation (LCT) known as the free metaplectic transform (FMT) was first studied in [1] by Folland and has gained much popularity in recent years as it can be used efficiently in many fields such as pattern recognition, filter design, optics, multidimensional signal processing, and analyzing the propagation of electromagnetic waves [2–5]. The FMT is generated by a general $2n \times 2n$ real, free symplectic matrix $\mathbf{M} = (A, B : C, D)$ with $n(2n+1)$ degrees of freedom [6]. The FMT embodies several time-frequency tools ranging from the classical Fourier, Fresnel transform, and even the fundamental operations of quadratic phase factor multiplication [7–9]. Mathematically, for any signal $f \in L^2(\mathbb{R}^n)$, the FMT of f with respect to the real free symplectic matrix $\mathbf{M} = (A, B : C, D)$ is given by earlier studies [8, 10]

 © 2023 John Wiley & Sons Ltd.