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Construction of biorthogonal wavelet packets on local fields of positive characteristic

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Abstract: Orthogonal wavelet packets lack symmetry, which is a much desired property in image and signal processing. The biorthogonal wavelet packets achieve symmetry where the orthogonality is replaced by biorthogonality. In the present paper, we construct biorthogonal wavelet packets on local fields of positive characteristic and investigate their properties by means of Fourier transforms. We also show how to obtain several new Riesz bases of the space $L^2(K)$ by constructing a series of subspaces of these wavelet packets. Finally, we provide algorithms for the decomposition and reconstruction using these biorthogonal wavelet packets.

Key words: Wavelet, multiresolution analysis, scaling function, wavelet packet, Riesz basis, local field, Fourier transform

1. Introduction

A field K equipped with a topology is called a *local field* if both the additive K^+ and multiplicative groups K^* of K are locally compact Abelian groups. The local fields are essentially of two types: zero and positive characteristic (excluding the connected local fields $\mathbb{R}$ and $\mathbb{C}$). Examples of local fields of characteristic zero include the p-adic field $\mathbb{Q}_p$ where as local fields of positive characteristic are the Cantor dyadic group and the Vilenkin p-groups. Even though the structures and metrics of local fields of zero and positive characteristics are similar, their wavelet and multiresolution analysis theory are quite different. In recent years, local fields have attracted the attention of several mathematicians, and have found innumerable applications not only in number theory but also in representation theory, division algebras, quadratic forms, and algebraic geometry. As a result, local fields are now consolidated as part of the standard repertoire of contemporary mathematics. For more about local fields and their applications, we refer to the monographs [15, 24].

In recent years there has been considerable interest in the problem of constructing wavelet bases on various groups, namely, Cantor dyadic groups [12], locally compact Abelian groups [9], p-adic fields [11], and Vilenkin groups [14]. Benedetto and Benedetto [3] developed a wavelet theory for local fields and related groups. They did not develop the multiresolution analysis (MRA) approach; their method is based on the theory of wavelet sets and only allows the construction of wavelet functions whose Fourier transforms are characteristic functions of some sets. The concept of multiresolution analysis on local fields of positive characteristic was introduced by Jiang et al. [10]. They pointed out a method for constructing orthogonal wavelets on local field K with a

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