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Original research article

Generalized wave packet transform based on convolution operator in the quaternion quadratic-phase Fourier domain

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ABSTRACT

The classical quaternion quadratic-phase Fourier transform fails in locating the quaternion quadratic-phase domain frequency contents that is required in numerous applications. In order to address this drawback, we in this paper proposed a novel transform coined as quaternion quadratic-phase wave packet transform (Q-QPWPT). The preliminary findings are the derivation of fundamental properties like linearity, parity, scaling, dilation and orthogonality relation. Moreover, some key harmonic analysis results like energy conservation, inversion formula and characterization of range are obtained. Besides, we also derived the Heisenberg's and logarithmic uncertainty principles. The crux of the paper lies in presenting an illustrative example and some potential applications.

1. Introduction

The quadratic-phase Fourier transform (QPFT) is the most recent generalization of the classical Fourier transform (FT) with five real parameters appeared via the way of reproducing kernels. It gives treatment to both the stationary and non-stationary signals in a simple fashion.

For a arbitrary real parameter set $\mathbb{m} = (A, B, C, D, E)$, $B \neq 0$ the quadratic-phase Fourier transform of any signal $f \in L^2(\mathbb{R})$ is defined by [1]

$$\mathcal{Q}_{\mathbb{m}}[f](w) = \int_{\mathbb{R}} f(x)\Omega_{\mathbb{m}}(x, w)dx, \tag{1.1}$$

where $\Omega_{\mathbb{m}}(x, w)$ represents quadratic-phase kernel given by

$$\Omega_{\mathbb{m}}(x, w) = \frac{1}{\sqrt{2\pi}} e^{-i(Ax^2+Bxw+Cw^2+Dx+Ew)}. \tag{1.2}$$

The inversion and Parseval's formulae corresponding to QPFT are given by

$$f(x) = \frac{B}{\sqrt{2\pi}} \int_{\mathbb{R}} \mathcal{Q}_{\mathbb{m}}[f](w)\overline{\Omega_{\mathbb{m}}(x, w)}dw, \tag{1.3}$$

and

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