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Quadratic-phase wave packet transform

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ABSTRACT

The quadratic-phase Fourier transform (QPFT) has gained much popularity in recent years because of its applications in image and signal processing. However, the QPFT is inadequate for localizing the quadratic-phase spectrum, which is required in some applications. In this paper, the quadratic-phase wave packet transform (QP-WPT) is proposed to address this problem, based on the wave packet transform (WPT) and QPFT. Firstly, we propose the definition of the QP-WPT and give its relation with windowed Fourier transform (WFT). Secondly, several notable inequalities and important properties of newly defined QP-WPT, such as boundedness, reconstruction formula, Moyal's formula, reproducing kernel are derived. Finally, we formulate several classes of uncertainty inequalities, such as Leib's uncertainty principle, logarithmic uncertainty inequality and the Heisenberg uncertainty inequality.

1. Introduction

The Fourier transform (FT) is an important tool in optical communication and signal processing [1]. However, owing to its global kernel the FT is incapable of obtaining information about local properties of the signal. But, the actual signals are often non-stationary or time-variable, so to overcome this problem, the short-time Fourier transform (STFT) is employed that uses a time window of fixed length applied at regular intervals so that we can obtain a portion of the signal considered to be stationary [2]. The resulting time-varying spectral depiction is critical for non-stationary signal analysis, but in this case it comes at fixed spectral and temporal resolution. The wavelet analysis [3,4] provides an attractive and pinch-hitting tool to the STFT by using an optical multichannel correlator with a bank of wavelet transform (WT) filters, which can provide a better illustration of the signal instead of the STFT. Nonetheless, in the high frequency region WT has poor frequency resolution. To solve this defect the wave packet transform (WPT) was proposed by combining the merits of STFT and WT [5,6]. WPT is a linear transform which uses the Weyl operator and the wave packages.

In recent years, researchers have successfully applied WPT in the fields of wireless communication, denoising, and image compression [7–14]. WPT is used widely in signal processing as it has some better morality than WT [15,16]. Moreover, it can realize multilevel decomposition and analyze the high frequency decomposition that is not achieved in traditional discrete WT. The frequency subbands of signal are selected via wave packet decomposition, that improves the time–frequency resolution capability

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