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Scaled Wigner distribution in the offset linear canonical domain

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ABSTRACT

The scaled Wigner distribution is obtained from the classical Wigner distribution by replacing the instantaneous auto-correlation with fractional instantaneous auto-correlation which is parameterized by a constant $k \in \mathbb{Q}^+$. In this paper, we introduce the scaled Wigner distribution in the offset linear canonical domain (SWDOLC). A natural magnification effect characterized by the extra degrees of freedom of the offset linear canonical transform (OLCT) and by a factor k on the frequency axis enables the SWDOLC to have flexibility so that it can be used in cross-term reduction. We initiate our investigation by studying the fundamental properties of the proposed transform, including the marginal, conjugate-symmetry, shifting, scaling, inverse and Moyal's formulae by using the machinery of SWDOLC and operator theory. Moreover, the convolution and correlation properties for the proposed transform are studied. The crux of the paper lies in the applications of SWDOLC for the detection of single-component and bi-component linear-frequency-modulated (LFM) signals.

1. Introduction

The Wigner distribution (WD) [1–7] is regarded as the most important distribution of all the time–frequency distributions. It is regarded as a versatile frequency analysis tool suitably meant for the analysis of time–frequency characteristics of non-transient signals, particularly the detection of chirp-like signals, such as the linear-frequency-modulated (LFM) signals which are frequently encountered in wireless communications, medical imaging sonar, radar and other modern day communication systems. For any finite energy signal $f(t)$ the WD is defined as [8,9]

$$WD_f(t,u) = \int_{\mathbb{R}} f\left(t+\frac{\tau}{2}\right) f^*\left(t-\frac{\tau}{2}\right) e^{-iu\tau} d\tau, \tag{1.1}$$

where $f\left(t+\frac{\tau}{2}\right) f^*\left(t-\frac{\tau}{2}\right)$ denotes the instantaneous auto-correlation function of the signal $f(t)$ (superscript $*$ denotes complex conjugate). It can be considered as a local spatial frequency spectrum of the signal, and has wide range of applications in wave optics, geometrical optics, Fourier optics, matrix optics, ray optics and radiometry [10]. In many practical applications such as optics, radar and sonar systems, we deal with a mono-component linear-frequency-modulated (LFM) signal and it is well-known that the Wigner distribution (WD) offers perfect localization (localized on a straight line) to these mono-component LFM signals. However, when applied to a multi-component (bi-component) case, cross terms appear due to quadratic nature of WD. Taking the general form of bi-component LFM signal i.e. $f(t) = f_1(t) + f_2(t)$ whose WD reads as:

$$WD_f(t,u) = WD_{f_1}(t,u) + WD_{f_2}(t,u) + WD_{f_1,f_2}(t,u) + WD_{f_2,f_1}(t,u), \tag{1.2}$$

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