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RESEARCH ARTICLE

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Clifford-valued linear canonical wavelet transform and the corresponding uncertainty principles

Shahbaz Rafiq | **Mohammad Younus Bhat**

Department of Mathematical Sciences, Islamic University of Science and Technology, Kashmir, Jammu and Kashmir, India

Correspondence
Mohammad Younus Bhat, Department of Mathematical Sciences, Islamic University of Science and Technology, Kashmir, Jammu and Kashmir 192122, India.
Email: gyounusg@gmail.com

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The present article establishes a novel transform known as Clifford-valued linear canonical wavelet transform which is intended to represent n-dimensional Clifford-valued signals at various scales, locations, and orientations. The suggested transform is capable of representing signals in the Clifford domain in addition to inheriting the characteristics of the Clifford wavelet transform. In the beginning, we demonstrate the proposed transform by the help of n-dimensional difference of Gaussian wavelets. We then establish the fundamental properties of the proposed transform like Parseval's formula, inversion formula, and characterization of its range using Clifford linear canonical transform and its convolution. To conclude our work, we derive an analog of Heisenberg's and local uncertainty inequalities for the proposed transform.

KEYWORDS
Clifford algebra, Clifford-valued linear canonical wavelet transform, Gaussian wavelets, Heisenberg's uncertainty inequality

MSC CLASSIFICATION
30G35, 15A66, 94A12, 43A32

1 | MOTIVATION AND INTRODUCTION

For analyzing signals and images, transforms like Fourier and wavelet are essential and effective tools. Using the Fourier transform, the signals from the original domain are mapped to frequency one. The fact is that the characteristics of a signal are more clearly visible in the frequency domain. On the other hand, wavelet bases signals are localized in both time and frequency domains [1] and thus provide a well-structured representation of signals and hence yield a better information about the behavior of the signals. It was Morlet et al. [2] who first worked on wavelet analysis to study seismic waves. He along with Grossman investigated a mathematical study of a continuous wavelet transform [3]. Meyer [4] identified the relation between harmonic analysis and Morlet's theory and provided a mathematical foundation to the continuous wavelet theory which lead to the development of wavelet analysis. The continuous wavelet transform of a square integrable function f starts by a convolution with copies of a given mother wavelet $\mathcal{T}$ translated and dilated by $b \in \mathbb{R}$ and $a \in \mathbb{R}^+$, respectively [1]. The function $\mathcal{T}$ is required to follow the following admissibility condition as

$$\mathcal{A}_{\mathcal{T}} = \int_{\mathbb{R}} \frac{|\widehat{\mathcal{T}(\mathbf{x})}|^2}{|\mathbf{x}|} d\mathbf{x} < +\infty \tag{1}$$

where $\widehat{\mathcal{T}(\mathbf{x})}$ is the Fourier transform of $\mathcal{T}$ [1]. One can visit [5] for more information on real wavelets.

Wavelet theory has shown remarkable success in several special domains like image/signal processing, statistics, finance, health, and engineering. This achievement motivate researchers to create new, more effective methods

