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DUAL WAVELETS ASSOCIATED WITH NONUNIFORM MRA

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Abstract. A generalization of Mallat's classical multiresolution analysis, based on the theory of spectral pairs, was considered in two articles by Gabardo and Nashed. In this setting, the associated translation set is no longer a discrete subgroup of $\mathbb{R}$ but a spectrum associated with a certain one-dimensional spectral pair and the associated dilation is an even positive integer related to the given spectral pair. In this paper, we construct dual wavelets which are associated with Nonuniform Multiresolution Analysis. We show that if the translates of the scaling functions of two multiresolution analyses are biorthogonal, then the associated wavelet families are also biorthogonal. Under mild assumptions on the scaling functions and the wavelets, we also show that the wavelets generate Riesz bases.

1. Introduction

Multiresolution analysis (MRA) is an important mathematical tool since it provides a natural framework for understanding and constructing discrete wavelet systems. A multiresolution analysis is an increasing family of closed subspaces $\{V_j : j \in \mathbb{Z}\}$ of $L^2(\mathbb{R})$ such that $\bigcap_{j\in\mathbb{Z}} V_j = \{0\}$, $\bigcup_{j\in\mathbb{Z}} V_j$ is dense in $L^2(\mathbb{R})$ and which satisfies $f \in V_j$ if and only if $f(2\cdot) \in V_{j+1}$. Furthermore, there exists an element $\varphi \in V_0$ such that the collection of integer translates of function φ, $\{\varphi(\cdot - k) : k \in \mathbb{Z}\}$ represents a complete orthonormal system for V_0. The function φ is called the *scaling function* or the *father wavelet*. The concept of multiresolution analysis has been extended in various ways in recent years. These concepts are generalized to $L^2(\mathbb{R}^d)$, to lattices different from $\mathbb{Z}^d$, allowing the subspaces of multiresolution analysis to be generated by Riesz basis instead of orthonormal basis, admitting a finite number of scaling functions, replacing the dilation factor 2 by an integer $M \geq 2$ or by an expansive matrix $A \in GL_d(\mathbb{R})$ as long as $A \subset A\mathbb{Z}^d$. But in all these cases, the translation set is always a group. Recently, Gabardo and Nashed in [9] defined a multiresolution analysis associated with a translation set $\{0, r/N\} + 2\mathbb{Z}$, where $N \geq 1$ is an integer, $1 \leq r \leq 2N-1$, r is an odd integer and r, N are relatively prime, a

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