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Generalized inequalities for nonuniform wavelet frames in linear canonical transform domain

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Abstract. A constructive algorithm based on the theory of spectral pairs for constructing nonuniform wavelet basis in $L^2(\mathbb{R})$ was considered by Gabardo and Nashed. In this setting, the associated translation set is a spectrum Λ which is not necessarily a group nor a uniform discrete set, given $\Lambda = \{0, r/N\} + 2\mathbb{Z}$, where $N \geq 1$ (an integer) and r is an odd integer with $1 \leq r \leq 2N-1$ such that r and N are relatively prime and $\mathbb{Z}$ is the set of all integers. In this article, we continue this study based on non-standard setting and obtain some inequalities for the nonuniform wavelet system $\left\{\mathfrak{f}^{M}_{j,\lambda}(x) = (2N)^{j/2}\mathfrak{f}\left((2N)^j x - \lambda\right)e^{-\frac{i\pi A}{B}(t^2-\lambda^2)}, j \in \mathbb{Z}, \lambda \in \Lambda\right\}$ to be a frame associated with linear canonical transform in $L^2(\mathbb{R})$. We use the concept of linear canonical transform so that our results generalise and sharpen some well-known wavelet inequalities.

1. Introduction

Frames were widely studied by Duffin and Schaeffer [9] in the light of non-harmonic Fourier series in year 1952. They were further investigated in 1986 by Daubechies et al.[7]. This process of frame study continued. The unique and interesting properties of frames and their duals make them able to play an important role in the characterization of signal and image processing, Image processing, signal spaces, sampling theory and many other fields. To be precise, we can say a frame is a collection of functions or signals in a separable Hilbert space which allows stable but not unique decomposition. Mathematically, we can say a family $\{f_k\}_{k=1}^{\infty}$ of functions of a Hilbert space $\mathbb{H}$ is called a *frame* for $\mathbb{H}$ if we can find constant $A, B > 0$ with the condition $f \in \mathbb{H}$,

$$A\|f\|_2^2 \leq \sum_{k=1}^{\infty} \left|\left\langle f, f_k\right\rangle\right|^2 \leq B\|f\|_2^2, \tag{1}$$

where A is lower frame bound while as B is the upper frame bound. When the bounds are equal, we have a *tight frame*. We have a *normalized tight frame* when $A = B = 1$. The frames which are born with the joint action of dilations and translations of finite number of signals will worth to study. To investigate these frames, for $a, b \in \mathbb{R}$ with $a > 1$, and $b > 0$, we define the wavelet systems as

$$\mathcal{F}(\mathfrak{f}, a, b) = \left\{\mathfrak{f}_{j,k} =: a^{j/2}\mathfrak{f}\left(a^j x - kb\right) : j, k \in \mathbb{Z}\right\}. \tag{2}$$

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