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Vector-valued nonuniform multiresolution analysis associated with linear canonical transform domain

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Abstract. A generalization of Mallat's classical multiresolution analysis, based on the theory of spectral pairs, was considered in two articles by Gabardo and Nashed. In this setting, the associated translation set is no longer a discrete subgroup of $\mathbb{R}$ but a spectrum associated with a certain one-dimensional spectral pair and the associated dilation is an even positive integer related to the given spectral pair. In this paper, we continue the study based on this nonstandard setting and introduce vector-valued nonuniform multiresolution analysis associated with linear canonical transform (LCT-VNUMRA) where the associated subspace V_0^{φ} of the function space $L^2\left(\mathbb{R}, \mathbb{C}^M\right)$ has an orthonormal basis of the form $\left\{\Phi(x-\lambda)e^{-\frac{i\pi A}{B}(t^2-\lambda^2)}\right\}_{\lambda\in\Lambda}$ where $\Lambda = \{0, r/N\} + 2\mathbb{Z}, N \geq 1$ is an integer and r is an odd integer such that r and N are relatively prime. We establish a necessary and sufficient condition for the existence of associated wavelets and derive an algorithm for the construction of vector-valued nonuniform multiresolution analysis starting from a vector refinement mask with appropriate conditions

1. Introduction

Multiresolution analysis (MRA) is an important mathematical tool since it provides a natural framework for understanding and constructing discrete wavelet systems. An MRA is an increasing family of closed subspaces $\left\{V_j : j \in \mathbb{Z}\right\}$ of $L^2(\mathbb{R})$ such that $\bigcap_{j\in\mathbb{Z}} V_j = \{0\}$, $\bigcup_{j\in\mathbb{Z}} V_j$ is dense in $L^2(\mathbb{R})$ and which satisfies $f \in V_j$ if and only if $f(2\cdot) \in V_{j+1}$. Furthermore, there exists an element $\varphi \in V_0$ such that the collection of integer translates of function φ, $\{\varphi(\cdot - k) : k \in \mathbb{Z}\}$ represents a complete orthonormal system for V_0. The function φ is called the *scaling function* or the *father wavelet*. The concept of MRA has been extended in various ways in recent years. These concepts are generalized to $L^2\left(\mathbb{R}^d\right)$, to lattices different from $\mathbb{Z}^d$, allowing the subspaces of MRA to be generated by Riesz basis instead of orthonormal basis, admitting a finite number of scaling functions, replacing the dilation factor 2 by an integer $M \geq 2$ or by an expansive matrix $A \in GL_d(\mathbb{R})$ as long as $A \subset A\mathbb{Z}^d$. On the other hand, Xia and Suter [20] introduced the concept of vector-valued MRA and orthogonal vector-valued wavelet basis and showed that vector-valued wavelets are a class of generalized multiwavelets. Chen and Cheng [11] presented the construction of a class of compactly supported orthogonal vector-valued wavelets and investigated the properties of vector-valued wavelet packets. Vector-valued wavelets are a class of generalized multiwavelets and multiwavelets can be generated from the component function in vector-valued wavelets. Vector-valued wavelets and

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