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Towards quaternion quadratic-phase Fourier transform

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The quadratic-phase Fourier transform (QPFT) is a neoteric addition to the class of integral transforms and embodies a variety of signal processing tools like the Fourier, fractional Fourier, linear canonical, and special affine Fourier transform. In this paper, we generalize the quadratic-phase Fourier transform to quaternion-valued signals, known as the quaternion quadratic-phase Fourier transform (Q-QPFT). We initiate our investigation by studying the QPFT of 2D quaternionic signals, and later on, we introduce the Q-QPFT of 2D quaternionic signals. Using the fundamental relationship between the Q-QPFT and quaternion Fourier transform (QFT), we derive the inversion, Parseval's, and Plancherel's formulae associated with the Q-QPFT. Some other properties including linearity, shift, and modulation of the Q-QPFT are also studied. Finally, we formulate several classes of uncertainty principles (UPs) for the Q-QPFT like Heisenberg-type UP, logarithmic UP, Hardy's UP, Beurling's UP, and Donoho–Stark's UP. This study can be regarded as the first step in the applications of the Q-QPFT in the real world.

KEYWORDS
Donoho–Stark, inversion, modulation, Parseval's formula, quaternion quadratic-phase Fourier transform, uncertainty principle

MSC CLASSIFICATION
42C40, 42A38, 94A12, 30G30

1 | INTRODUCTION

In the world of time-frequency analysis, the most recent and important signal processing tool is the quadratic-phase Fourier transform (QPFT). It was introduced by Castro et al. [1] It provides a unified treatment of both the transient and non-transient signals in a simple and insightful fashion. The QPFT has five real parameters with exponential kernel. With a slight modification in Castro et al. [1], we define the QPFT as

$$Q_\mu[f](w) = \int_{\mathbb{R}} f(x)\Lambda_\mu(x, w)dx, \tag{1}$$

where $\Lambda_\mu(x, w)$ is a quadratic-phase kernel given by

$$\Lambda_\mu(x, w) = \sqrt{\frac{bi}{2\pi}}e^{-i(ax^2+bxw+cw^2+dx+ew)}, \tag{2}$$

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