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ORIGINAL PAPER

Convolution and correlation theorems for Wigner–Ville distribution associated with the quaternion offset linear canonical transform

Convolution and correlation theorems for WVD associated with the QOLCT

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Abstract
The quaternion offset linear canonical transform (QOLCT) has gained much popularity in recent years because of its applications in many areas, including image and signal processing. At the same time, the applications of Wigner–Ville distribution (WVD) in signal analysis and image processing cannot be excluded. In this paper, we investigate the Wigner–Ville distribution associated with quaternion offset linear canonical transform (WVD-QOLCT). Firstly, we propose the definition of the WVD-QOLCT, and then, several important properties of newly defined WVD-QOLCT, such as reconstruction formula, orthogonality relation, are derived. Secondly, a novel canonical convolution operator and a related correlation operator for WVD-QOLCT are proposed. Based on the proposed operators, the corresponding generalized convolution and correlation theorems are studied. Moreover on the application part, detection of the linear frequency modulated signals is established in detail by constructing an example.

Keywords Quaternion algebra · Offset linear canonical transform · Quaternion offset linear canonical transform · Wigner–Ville distribution · Convolution · Correlation · Modulation

Mathematics Subject Classification 11R52 · 42C40 · 42C30 · 43A30

1 Introduction

In the time–frequency signal analysis, the classical Wigner–Ville distribution (WVD) or Wigner–Ville transform (WVT) has an important role to play. Eugene Wigner introduced the concept WVD while making his calculation of the quantum corrections. Later on, it was J. Ville who derived it independently as a quadratic representation of the local time–frequency energy of a signal in 1948. Numerous important properties of WVT have been studied by many authors. On replacing the kernel of the classical Fourier transform (FT) with the kernel of the LCT in the WVD domain, this transform can be extended to the domain of linear canonical transform [3–6,13–19].

On the other hand, the quaternion Fourier transform (QFT) is of the interest in the present era. Many important properties like shift, modulation, convolution, correlation, differentiation, energy conservation, uncertainty principle of QFT have been found. Many generalized transforms are closely related to the QFTs, for example, the quaternion wavelet transform, fractional quaternion Fourier transform, quaternion linear canonical transform, and quaternionic windowed Fourier transform. Based on the QFTs, one may also extend the WVD to the quaternion algebra while enjoying similar properties as in the classical case. Many authors generalized the classical WVD to quaternion algebra, which they called as the quaternion Wigner–Ville distribution (QWVD). For more details, we refer to [1,2,7–12].

Moving to the side of generalizations linear canonical transform (LCT), it is here worth mentioning that the LCT with four parameters (a, b, c, d) has been generalized to a six-parameter transform (a, b, c, d, u_0, w_0) known as offset linear canonical transform (OLCT). Due to the time shift-

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