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Convolution based quadratic-phase Stockwell transform: theory and uncertainty relations

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Abstract
In the field of optics and signal processing, the novel quadratic-phase Fourier transform (QPFT) has emerged as a powerful tool. However, it has a drawback as it fails in locating the quadratic-phase domain frequency contents which is much needed in numerous applications, where a joint information of time and quadratic-phase Fourier domain frequency is needed. On the other hand the Stockwell transform (ST), which is a time-frequency representation tool known for its local spectral phase properties in signal processing, uniquely combines elements of wavelet transforms and the short-time Fourier transform (STFT). However, its signal analysis ability is restricted in the time-frequency plane. In order to rectify the limitations of the QPFT and the ST, we in this paper proposed the quadratic-phase Stockwell transform (QPST) by employing the convolution structure of quadratic-phase Fourier transforms. Firstly, we examined the resolution of the QPST in the time and QPFT domains and then derived some of its basic properties, such as Rayleigh's energy theorem, inversion formula and characterization of the range. Besides, we also derived a direct relationship between the well-known Wigner-Ville distribution and the proposed QPST. In the sequel, we introduced discrete version of the QPST, together with its reconstruction formula. Further , we are successful in establishing the well known Heisenberg's and logarithmic uncertainty principles associated with the proposed QPST. Towards the culmination of this paper, we presented applications of the proposed QPST in detection of chirp signals in presence of echo. The simulation results lucidly demonstrate that the QPST performs exceptionally well in comparison with the conventional ST.

Keywords Stockwell transform · Wavelet transform · Wigner distribution · Quadratic-phase Fourier transform · Time-frequency analysis · Uncertainty principle

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