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On the nonhomogeneous wavelet bi-frames for reducing subspaces of $H^s(K)$

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Abstract. Ahmad and Shiekh in *Filomat* 34: 6(2020), have constructed dual wavelet frames in Sobolev spaces on local fields of positive characteristic. We continued the study and provided the characterization of nonhomogeneous wavelet bi-frames. First of all we introduce the reducing subspaces of Sobolev spaces over local fields of prime characteristics and then provide the way to characterize the nonhomogeneous wavelet bi-frames over such fields. Our results are better than those established by Ahmad and Shiekh.

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1. Introduction

To start with it is to be noted that a refinable structure gives birth to the classical nonhomogeneous systems with some technical restrictions on them [16, 22, 23, 24]. The wavelet systems thus obtained have fast wavelet transform. However the correspondence between them is not exact. It was Han [22, 23] who showed that the nonstationary wavelets and nonhomogeneous wavelet systems are closely related. With these considerations in mind, our aim in this paper is to construct and characterize nonhomogeneous wavelet bi-frames (NWBFs) on Sobolev spaces over local fields of positive characteristic.

Moving to the side of frames it is here worth to mention that Duffin and Schaeffer [21] introduced frames in non-harmonic Fourier series in 1952. They were again studied in 1986 by Daubechies and the process continued. Frames and the dual frames have an important role to play in the characterization of signal, image and video processing, function spaces, sampling theory and many more. Mathematically a frame is defined in the following manner. A sequence of functions $\{f_k\}_{k=1}^{\infty}$ of Hilbert space $\mathbb{H}$ is called a *frame* for $\mathbb{H}$ if there exist constants $\mathfrak{A}, \mathfrak{B} > 0$ such that for all $f \in \mathbb{H}$,

$$\mathfrak{A}\|f\|_2^2 \leq \sum_{k=1}^{\infty} \left|\langle f, f_k\rangle\right|^2 \leq \mathfrak{B}\|f\|_2^2,$$

where $\mathfrak{A}$ is lower bound and $\mathfrak{B}$ is the upper one. If $\mathfrak{A} = \mathfrak{B}$, then we have a *tight frame*. If $\mathfrak{A} = \mathfrak{B} = 1$, then we end up with a *normalized tight frame*. For more about frames, we refer to [19, 18, 20] and the references therein.

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