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ORIGINAL RESEARCH

Two-dimensional quaternion linear canonical transform: a novel framework for probability modeling

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Abstract

In this paper, we present the two-dimensional quaternion linear canonical transform (2D QLCT) as a novel tool for probability modeling, specifically designed to enhance the analysis of multi-dimensional and complex-valued signals. We develop a comprehensive framework that extends classical probability theory by leveraging the properties of 2D QLCT, providing new insights into probability distributions. The key contributions include the introduction of novel mathematical properties, derivation of the characteristic function in the 2D QLCT domain, and the development of covariance structures within this framework. These results not only broaden the theoretical foundations of probability theory but also offer potential applications across fields such as signal processing, engineering, and statistical analysis.

Keywords Two-dimensional quaternion linear canonical transform · Quaternion characteristic function · Quaternion probability density function · Quaternion cumulative distribution function · Expected value · Quaternion covariance

JEL Classification 46L53 · 42B10 · 42B05 · 60E05

1 Introduction

The linear canonical transform (LCT) is a powerful generalization of the Fourier transform (FT), widely utilized across various domains, including signal processing, optics, and quantum mechanics. The theory of the LCT emerged in the early 1970s via the foundational contributions of Collins Jr. in paraxial optics [1] and Moshinsky

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