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Multiwavelets on local fields of positive characteristic

Mohammad Younus Bhat

Abstract. Multiwavelets have more freedom in their construction and thus can combine more useful properties than the scalar wavelets. Symmetric scaling functions constructed have short support, generate an orthogonal MRA and provide approximation order 2. These properties are very desirable in many applications but cannot be achieved by one scaling function. In this paper we construct multiwavelets on local fields of positive characteristic. We also give their characterization by means of some basic equations in the frequency domain.

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1. Introduction

Multiresolution analysis (MRA) is an important mathematical tool since it provides a natural framework for understanding and constructing discrete wavelet systems. Usually it is assumed that an MRA is generated by one scaling function and dilates and translates of only one wavelet $\psi \in L^2(K)$ form a stable basis of $L^2(K)$. Here, we consider a generalization allowing several wavelet functions $\psi_1, \dots, \psi_r$. The vector generated from this system will be called as a multiwavelet on local fields of positive characteristic.

In recent years there has been a considerable interest in the problem of constructing wavelet bases on various groups, namely, Cantor dyadic groups,locally compact Abelian groups,6 positive half-line R^+, p-adic fields, Vilenkin groups, Heisenberg groups and Lie groups. Benedetto and Benedetto[1] developed a wavelet theory for local fields and related groups. They did not develop the MRA approach, their method is based on the theory of wavelet sets and only allows the construction of wavelet functions whose Fourier transforms are characteristic functions of some sets. Since local fields are essentially of two types: Zero and positive characteristic (excluding the connected local fields $\mathbb{R}$ and $\mathbb{C}$). Examples of local fields of characteristic zero include the p-adic field Q_p whereas local fields of positive characteristic are the Cantor dyadic group and the Vilenkin p-groups. Even though the structures and metrics of local fields of zero and positive characteristics are similar, but their wavelet and MRA theory are quite different. In recent years, local fields have attracted the attention of several mathematicians, and have found innumerable applications not only to number theory but also to representation theory, division algebras, quadratic forms and algebraic geometry. As a result, local fields are now consolidated as part of the standard repertoire of contemporary mathematics. For more details we refer to [1, 8, 9, 10] and the many references therein.

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