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Vector-valued nonuniform multiresolution analysis on local fields

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A multiresolution analysis (MRA) on local fields of positive characteristic was defined by Shah and Abdullah for which the translation set is a discrete set which is not a group. In this paper, we continue the study based on this nonstandard setting and introduce vector-valued nonuniform multiresolution analysis (VNUMRA) where the associated subspace V_0 of $L^2(K, \mathbb{C}^M)$ has an orthonormal basis of the form $\{\Phi(x-\lambda)\}_{\lambda\in\Lambda}$ where $\Lambda = \{0, r/N\} + \mathcal{Z}$, $N \geq 1$ is an integer and r is an odd integer such that r and N are relatively prime and $\mathcal{Z} = \{u(n) : n \in \mathbb{N}_0\}$. We establish a necessary and sufficient condition for the existence of associated wavelets and derive an algorithm for the construction of VNUMRA on local fields starting from a vector refinement mask $G(\xi)$ with appropriate conditions. Further, these results also hold for Cantor and Vilenkin groups.

Keywords: Nonuniform multiresolution analysis; local field; scaling function; vector-valued wavelets; Fourier transform.

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1. Introduction

Multiresolution analysis (MRA) is an important mathematical tool since it provides a natural framework for understanding and constructing discrete wavelet systems. A MRA is an increasing family of closed subspaces $\{V_j : j \in \mathbb{Z}\}$ of $L^2(\mathbb{R})$ such that $\bigcap_{j\in\mathbb{Z}} V_j = \{0\}$, $\bigcup_{j\in\mathbb{Z}} V_j$ is dense in $L^2(\mathbb{R})$ and which satisfies $f \in V_j$ if and only if $f(2\cdot) \in V_{j+1}$. Furthermore, there exists an element $\varphi \in V_0$ such that the collection of integer translates of function φ, $\{\varphi(\cdot - k) : k \in \mathbb{Z}\}$ represents a complete orthonormal