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Fractional vector-valued nonuniform MRA and associated wavelet packets on $L^2(\mathbb{R}, \mathbb{C}^M)$

M. Younus Bhat[1] · **Aamir H. Dar**[1]

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Abstract
A generalization of Mallat's classical multiresolution analysis, based on the theory of spectral pairs, was considered in two articles by Gabardo and Nashed. In this setting, the associated translation set is no longer a discrete subgroup of $\mathbb{R}$ but a spectrum associated with a certain one-dimensional spectral pair and the associated dilation is an even positive integer related to the given spectral pair. In this paper, we continue the study based on this nonstandard setting and introduce fractional vector-valued nonuniform multiresolution analysis (Fr-VNUMRA) where the associated subspace of the function space has an orthonormal basis. We establish a necessary and sufficient condition for the existence of associated wavelets and derive an algorithm for the construction of fractional vector-valued nonuniform multiresolution analysis starting from a vector refinement mask with appropriate conditions. Nevertheless, to extend the scope of the present study, we worked to construct the associated wavelet packets for such an MRA and investigate their properties by means of fractional Fourier transform.

Keywords Fractional non-uniform multiresolution analysis · Fractional Fourier transform · Scaling function · Wavelet packets

Mathematics Subject Classification 42C40 · 42C15 · 41A17 · 46F12 · 47G10

✉ M. Younus Bhat
gyounusg@gmail.com

Aamir H. Dar
adhkul740@gmail.com

[1] Department of Mathematical Sciences, Islamic University of Science and Technology, Awantipora, Pulwama, Jammu and Kashmir 192122, India

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