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**ORIGINAL RESEARCH**

# Nonhomogeneous Wavelet Bi-frames for Reducing Subspaces of $H^s(K)$ and their Characterization

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**Abstract** Bhat in *Annal. Univ. Craiova, Math. Comp. Scien. Series* 49: 2(2022), 401-410 has studied nonhomogeneous wavelet bi-frames in Sobolev spaces on local fields of positive characteristic. In this paper, we used the same platform to characterize nonhomogeneous wavelet bi-frames for the reducing subspaces of Sobolev spaces over local fields of positive characteristics.

**Keywords** Local field · Nonhomogeneous · Wavelet Bi-frames · Sobolev Spaces

**Mathematics Subject Classification** 42C40 · 42C15 · 43A70 · 11S85 · 47A25

## 1 Introduction

To begin with it is pertain to mention that a refinable structure generates the classical nonhomogeneous systems with some restrictions on them [16,22–24] which are technical in nature. The generated wavelet systems will have too fast wavelet transform. But the correspondence between these systems is not exact. Thanks to Han [22,23] who showed that the nonstationary wavelets and nonhomogeneous wavelet systems are related so closely. Later on tremendous work has been done nonhomogeneous wavelet systems. we refer to [29–34] and the references therein. Keeping in view these constructions, our aim in this paper is to characterize nonhomogeneous wavelet bi-frames (NWBFs) on Sobolev spaces over local fields of positive characteristic.

Let us now move to the frame theory. It were Duffin and Schaeffer [21] who introduced frames in non-harmonic Fourier series in 1952. The frame theory did not end up here. They were again visualized in 1986 by Daubechies and this process continues. Frames and the dual frames have an important role to play in the characterization of signal, image and video processing, sampling theory, function spaces and many other fields. From the mathematical point of view, a frame is defined as a sequence of functions $\{f_k\}_{k=1}^{\infty}$ of Hilbert space $\mathbb{H}$ if there exist constants $\mathcal{A}, \mathcal{B} > 0$ such that for all $f \in \mathbb{H}$,

$$\mathcal{A}\|f\|_2^2 \leq \sum_{k=1}^{\infty} |\langle f, f_k\rangle|^2 \leq \mathcal{B}\|f\|_2^2,$$

where $\mathcal{A}$ is lower bound and $\mathcal{B}$ is the upper one. If $\mathcal{A} = \mathcal{B}$, then we have a *tight frame*. If $\mathcal{A} = \mathcal{B} = 1$, then we end up with a *normalized tight frame*. For more details about frames, we refer to [18–20].

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