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VARIETIES OF PERMUTATIVE SEMIGROUPS CLOSED UNDER DOMINIONS

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ABSTRACT. In this paper, we partially generalize a result of Isbell from the class of commutative semigroups to some generalized class of commutative semigroups by showing that dominion of such semigroups belongs to the same class by using Isbell's zigzag theorem.

1. INTRODUCTION AND PRELIMINARIES

Let U be a subsemigroup of a semigroup S. Following Isbell [5], we say that U dominates an element d of S if for every semigroup T and for all homomorphisms $\beta, \gamma : S \longrightarrow T$ and $u\beta{=}u\gamma$ for every u in U implies $d\beta{=}d\gamma$. The set of all elements of S dominated by U is called dominion of U in S and we denote it by $Dom(U, S)$. It can be easily verified that $Dom(U, S)$ is a subsemigroup of S containing U.

The following theorem provided by Isbell [5], known as Isbell's zigzag theorem, is a most useful characterization of semigroup dominions and is of basic importance to our investigations.

Theorem 1.1. *([5], Theorem 2.3) Let U be a subsemigroup of a semigroup S and let $d \in S$. Then $d \in Dom(U, S)$ if and only if $d \in U$ or there exists a series of factorizations of d as follows:*

$$d = a_0t_1 = y_1a_1t_1 = y_1a_2t_2 = y_2a_3t_2 = \cdots = y_ma_{2m-1}t_m = y_ma_{2m} \quad (1.1)$$

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