Optik - International Journal for Light and Electron Optics 272 (2023) 170213

Contents lists available at ScienceDirect

Optik - International Journal for Light and Electron Optics

journal homepage: www.elsevier.com/locate/ijleo

Original research article

Wigner distribution and associated uncertainty principles in the framework of octonion linear canonical transform

Aamir H. Dar[1], M. Younus Bhat *,[1]

Department of Mathematical Sciences, Islamic University of Science and Technology, Kashmir, India

ARTICLE INFO

MSC:
42B10
46S10
94A12
42A38
30G30

Keywords:
Octonion linear canonical transform(OLCT)
Wigner distribution (WD)
Rayleigh's theorem
Riemann–Lebesgue Lemma
Uncertainty principle (UP)

ABSTRACT

The most recent generalization of octonion Fourier transform (OFT) is the octonion linear canonical transform (OLCT) that has become popular in present era due to its applications in color image and signal processing. On the other hand the applications of Wigner distribution (WD) in signal and image analysis cannot be excluded. In this paper, we introduce novel integral transform coined as the Wigner distribution in the octonion linear canonical transform domain (WDOL). We first propose the definition of the one dimensional WDOL (1D-WDOL), we extend its relationship with 1D-OLCT and 1D-OFT. Then explore several important properties of 1D-WDOL, such as reconstruction formula, Rayleigh's theorem. Second, we introduce the definition of three dimensional WDOL (3D-WDOL) and establish its relationships with the WD associated with quaternion LCT (WD-QLCT) and 3D-WD in LCT domain (3D-WDLCT). Then we study properties like reconstruction formula, Rayleigh's theorem and Riemann–Lebesgue Lemma associated with 3D-WDOL. The crux of this paper lies in developing well known uncertainty principles (UPs) including Heisenberg's UP, Logarithmic UP and Hausdorff–Young inequality associated with WDOL.

1. Introduction

Of all the time–frequency distributions, Wigner distribution (WD) [1–12] is regarded as the most important distribution. WD is considered as an important frequency analysis tool that is more suitable for the analysis of time–frequency characteristics of chirp-like signals, such as the linear-frequency-modulated (LFM) signals that are frequently used in wireless communications, medical imaging sonar, radar and many more. For any finite energy signals f and g the WD is defined as [13,14]

$$\mathcal{W}_{f,g}(t,w) = \int_{\mathbb{R}} f\left(t+\frac{x}{2}\right) g^*\left(t-\frac{x}{2}\right) e^{-iwx}dx, \tag{1.1}$$

where $f\left(t+\frac{x}{2}\right) g^*\left(t-\frac{x}{2}\right)$ represents the instantaneous auto-correlation relation of the signal $f(x)$. It is also viewed as a local spatial frequency spectrum of the signal, with tremendous applications in optics, matrix optics, wave optics, geometrical optics, Fourier ray optics and radiometry [15].

In the last few decades, the researcher's community has shown greater interest in the study of multidimensional hyper-complex signals defined by means of Cayley–Dickson algebras and there applications in image filtering, watermarking, color and image processing, edge detection and pattern recognition [16–21]. The Cayley–Dickson algebra of order 4 is known as quaternions. In quaternionic analysis, the quaternion Fourier transform (QFT) is the most basic and important time–frequency analysis tool for

* Corresponding author.
E-mail addresses: ahdkul740@gmail.com (A.H. Dar), gyounusg@gmail.com (M.Y. Bhat).

[1] Both the authors equally contributed towards this work.

https://doi.org/10.1016/j.ijleo.2022.170213
Received 17 September 2022; Accepted 7 November 2022
Available online 12 November 2022
0030-4026/