



Nonuniform Wavelet Packets on Local Fields of Positive Characteristic

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Abstract. The concept of nonuniform multiresolution analysis on local field of positive characteristic was considered by Shah and Abdullah for which the translation set is a discrete set which is not a group. We construct the associated wavelet packets for such an MRA and investigate their properties by means of the Fourier transform.

1. Introduction

Multiresolution analysis (MRA) is an important mathematical tool since it provides a natural framework for understanding and constructing discrete wavelet systems. A multiresolution analysis is an increasing family of closed subspaces $\{V_j : j \in \mathbb{Z}\}$ of $L^2(\mathbb{R})$ such that $\bigcap_{j \in \mathbb{Z}} V_j = \{0\}$, $\bigcup_{j \in \mathbb{Z}} V_j$ is dense in $L^2(\mathbb{R})$ and which satisfies $f \in V_j$ if and only if $f(2 \cdot) \in V_{j+1}$. Furthermore, there exists an element $\varphi \in V_0$ such that the collection of integer translates of function φ , $\{\varphi(\cdot - k) : k \in \mathbb{Z}\}$ represents a complete orthonormal system for V_0 . The function φ is called the *scaling function* or the *father wavelet*. The concept of multiresolution analysis has been extended in various ways in recent years. These concepts are generalized to $L^2(\mathbb{R}^d)$, to lattices different from \mathbb{Z}^d , allowing the subspaces of multiresolution analysis to be generated by Riesz basis instead of orthonormal basis, admitting a finite number of scaling functions, replacing the dilation factor 2 by an integer $M \geq 2$ or by an expansive matrix $A \in GL_d(\mathbb{R})$ as long as $A \in A\mathbb{Z}^d$. For more about wavelets and their applications, we refer the monograph [6].

In recent years there has been a considerable interest in the problem of constructing wavelet bases on various groups, namely, Cantor dyadic groups [10], locally compact Abelian groups [7], p -adic fields [9] and Vilenkin groups [11]. Recently, R. L. Benedetto and J. J. Benedetto [2] developed a wavelet theory for local fields and related groups. They did not develop the multiresolution analysis (MRA) approach, their method is based on the theory of wavelet sets and only allows the construction of wavelet functions whose Fourier transforms are characteristic functions of some sets. Since local fields are essentially of two types: zero and positive characteristic (excluding the connected local fields \mathbb{R} and \mathbb{C}). Examples of local

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