Research Article

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Semi-orthogonal wavelet frames on local fields

Abstract: We investigate semi-orthogonal wavelet frames on local fields of positive characteristic and provide a characterization of frame wavelets by means of some basic equations in the frequency domain. The theory of frame multiresolution analysis recently proposed by Shah [20] on local fields is used to establish equivalent conditions for a finite number of functions $\psi_1, \psi_2, \dots, \psi_L$ in $L^2(K)$ to generate a semi-orthogonal wavelet frame for $L^2(K)$.

Keywords: Frame multiresolution analysis, wavelet frame, semi-orthogonality, local field, Fourier transform

MSC 2010: 42C40, 42C15, 43A70, 11S85

DOI: 10.1515/anly-2015-0026
Received June 27, 2015; revised August 18, 2015; accepted August 29, 2015

1 Introduction

The notion of frames was first introduced by Duffin and Schaeffer [10] in the context of non-harmonic Fourier series. In signal processing, this concept has become very useful in analyzing the completeness and stability of linear discrete signal representations. Outside signal processing, frames did not seem to generate much interest until the seminal work by Daubechies, Grossmann and Meyer [8]. They combined the theory of continuous wavelet transforms with the theory of frames to introduce wavelet frames for $L^2(\mathbb{R})$. A wavelet frame is a generalization of an orthonormal wavelet basis by introducing redundancy into a wavelet system. By sacrificing orthonormality and allowing redundancy, wavelet frames become much easier to construct than the orthonormal wavelets. Wavelet frames have many properties that make them useful in the study of function spaces, signal and image processing, filter banks, wireless communications, etc. For more about wavelet frames and their applications, the reader is referred to [7, 9, 11, 12] and many references therein.

On the other hand, the most efficient way to construct an orthonormal wavelet is to construct it from an orthonormal multiresolution analysis (MRA). Since the use of an MRA has proven to be a very efficient tool in wavelet theory mainly because of its simplicity, it is of interest to try to generalize this notion as much as possible while preserving its connection with wavelet analysis. In this connection, Benedetto and Li [4] considered the dyadic *semi-orthogonal frame multiresolution analysis* of $L^2(\mathbb{R})$ with a single scaling function and successfully applied the theory in the analysis of narrow band signals. The characterization of the dyadic semi-orthogonal frame multiresolution analysis with a single scaling function admitting a single frame wavelet whose dyadic dilations of the integer translates form a frame for $L^2(\mathbb{R})$ was obtained independently by Benedetto and Treiber [5] by a direct method, and H. O. Kim, R. Y. Kim and Lim [16] using the theory of shift-invariant spaces. Later on, Xiaojiang [26] extended the results of Benedetto and Li's theory of frame multiresolution analysis to higher dimensions with arbitrary expansive matrix dilations, and established the necessary and sufficient conditions to characterize semi-orthogonal multiresolution analysis frames for $L^2(\mathbb{R}^n)$.

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