Afrika Matematika (2021) 32:745–756
https://doi.org/10.1007/s13370-020-00858-2

Bounds for the skew Laplacian energy of weighted digraphs

Bilal A. Chat[1] · **Hilal A. Ganie**[2] · **S. Pirzada**[3]

Received: 1 April 2020 / Accepted: 23 November 2020 / Published online: 4 January 2021

Abstract
Let $\mathbb{D}$ be a simple connected digraph with n vertices and m arcs and let $W(\mathbb{D}) = (\mathbb{D}, \omega)$ be the weighted digraph corresponding to $\mathbb{D}$, where the weights are taken from the set of non-zero real numbers. In this paper, we define the skew Laplacian matrix $SL(W(\mathbb{D}))$ and skew Laplacian energy $SLE(W(\mathbb{D}))$ of a weighted digraph $W(\mathbb{D})$, which is defined as the sum of the absolute values of the skew Laplacian eigenvalues, that is, $SLE(W(\mathbb{D})) = \sum_{i=1}^{n} |\rho_i|$, where $\rho_1, \rho_2, \ldots, \rho_n$ are the skew Laplacian eigenvalues of $W(\mathbb{D})$. We show the existence of the real skew Laplacian eigenvalues of a weighted digraph when the weighted digraph has an independent set such that all the vertices in the independent set have the same out-neighbors and in-neighbors. We obtain a Koolen type upper bound for $SLE(W(\mathbb{D}))$. Further, for a connected weighted digraph $W(\mathbb{D})$, we obtain bounds for $SLE(W(\mathbb{D}))$, in terms of different digraph parameters associated with the digraph structure $\mathbb{D}$. We characterize the extremal weighted digraphs attaining these bounds.

Keywords Weighted adjacency matrix (spectrum) · Skew Laplacian matrix (spectrum) · Eulerian weighted digraph · Skew Laplacian energy

Mathematics Subject Classification Primary: 05C50, 05C12 · Secondary: 15A18

✉ Bilal A. Chat
bchat1118@gmail.com

Hilal A. Ganie
hilahmad1119kt@gmail.com

S. Pirzada
pirzadasd@kashmiruniversity.ac.in

[1] Department of Mathematical Sciences, Islamic University of Science and Technology, Awantipora, Pulwama, Kashmir, India

[2] Department of School Education, JK Govt, Srinagar, Kashmir, India

[3] Department of Mathematics, University of Kashmir, Srinagar, India