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On skew Laplacian energy of directed graphs

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Abstract
Let $\mathscr{D}$ be a simple digraph with n-vertices, m arcs having skew Laplacian eigenvalues $\nu_1, \nu_2, \ldots, \nu_{n-1}, \nu_n = 0$. The skew Laplacian energy $SLE(\mathscr{D})$ of a digraph $\mathscr{D}$ is defined as $SLE(\mathscr{D}) = \sum_{i=1}^{n} |\nu_i|$. In this paper, we obtain the characteristic polynomial of skew Laplacian matrix of the digraph $\mathscr{D}_1 \rightarrow \mathscr{D}_2$ and also obtain the $SLE(\mathscr{D}_1 \rightarrow \mathscr{D}_2)$ in terms of $SLE(\mathscr{D}_1)$ and $SLE(\mathscr{D}_2)$ and show the existence of some families of skew Laplacian equienergetic digraphs.

Keywords Digraphs · Skew Laplacian matrix · Skew Laplacian spectrum · Skew Laplacian energy

Mathematics Subject Classification Primary 05C50 · 05C12; Secondary 05C30 · 15A18

1 Introduction

Let $\mathscr{D}$ be a simple digraph with n vertices $v_1, v_2, \ldots, v_n$ and m arcs. Let $d_i^+ = d^+(v_i),\ d_i^- = d^-(v_i)$ and $d_i = d_i^+ + d_i^-,\ i = 1, 2, \ldots, n$ be the out-degree, in-degree and degree of the vertices of $\mathscr{D}$, respectively. The out-adjacency matrix $A^+(\mathscr{D}) = (a_{ij})$ of a digraph $\mathscr{D}$ is the $n \times n$ matrix, where $a_{ij} = 1$, if (v_i, v_j) is an arc and $a_{ij} = 0$, otherwise. The in-adjacency matrix $A^-(\mathscr{D}) = (a_{ij})$ of a digraph $\mathscr{D}$ is the $n \times n$ matrix, where: $a_{ij} = 1$, if (v_j, v_i) is an arc and $a_{ij} = 0$, otherwise. It is clear that $A^-(\mathscr{D}) = (A^+(\mathscr{D}))^t$.

The skew adjacency matrix $S(\mathscr{D}) = (s_{ij})$ of a digraph $\mathscr{D}$ is the $n \times n$ matrix, where

$$s_{ij} = \begin{cases} 1, & \text{if there is an arc from } v_i \text{ to } v_j, \\ -1, & \text{if there is an arc from } v_j \text{ to } v_i, \\ 0, & \text{otherwise.} \end{cases}$$

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