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ON SKEW LAPLACIAN SPECTRA AND SKEW LAPLACIAN ENERGY OF DIGRAPHS

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Abstract. Let $\mathscr{D}$ be a simple digraph with n vertices, m arcs having skew Laplacian eigenvalues $\nu_1, \nu_2, \ldots, \nu_{n-1}, \nu_n = 0$. The skew Laplacian energy $SLE(\mathscr{D})$ of a digraph $\mathscr{D}$ is defined as $SLE(\mathscr{D}) = \sum_{i=1}^{n} |\nu_i|$. We obtain upper and lower bounds for $SLE(\mathscr{D})$, which improves some previously known bounds. We also show that every even positive integer is indeed the skew Laplacian energy of some digraph.

1. Introduction

Let $\mathscr{D}$ be a simple digraph with n vertices $v_1, v_2, \ldots, v_n$ and m arcs. Let $d_i^+ = d^+(v_i)$, $d_i^- = d^-(v_i)$ and $d_i = d_i^+ + d_i^-$, $i = 1, 2, \ldots, n$ be respectively, the out-degree, in-degree and degree of the vertices of the digraph $\mathscr{D}$. The out-adjacency matrix $A^+(\mathscr{D}) = (a_{ij})$ of a digraph $\mathscr{D}$ is the $n \times n$ matrix, where $a_{ij} = 1$, if (v_i, v_j) is an arc and $a_{ij} = 0$, otherwise. The in-adjacency matrix $A^-(\mathscr{D}) = (a_{ij})$ of a digraph $\mathscr{D}$ is the $n \times n$ matrix, where $a_{ij} = 1$, if (v_j, v_i) is an arc and $a_{ij} = 0$, otherwise. It is clear that $A^-(\mathscr{D}) = (A^+(\mathscr{D}))^t$.

The skew adjacency matrix $S(\mathscr{D}) = (s_{ij})$ of a digraph $\mathscr{D}$ is the $n \times n$ matrix, where

$$s_{ij} = \begin{cases} 1, & \text{if there is an arc from } v_i \text{ to } v_j, \\ -1, & \text{if there is an arc from } v_j \text{ to } v_i, \\ 0, & \text{otherwise.} \end{cases}$$

It is clear that $S(\mathscr{D})$ is a skew symmetric matrix, so all its eigenvalues are zero or purely imaginary. The energy of the matrix $S(\mathscr{D})$ was considered in [1], and is defined

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