On Potentially Graphical Sequences of G - E(H)

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Abstract. A loopless graph on *n* vertices in which vertices are connected at least by *a* and at most by *b* edges is called a (a,b,n)-graph. A (b,b,n)-graph is called (b,n)-graph and is denoted by K_n^b (it is a complete graph), its complement by \overline{K}_n^b . A non increasing sequence $\pi = (d_1, \dots, d_n)$ of nonnegative integers is said to be (a,b,n) graphic if it is realizable by an (a,b,n)-graph. We say a simple graphic sequence $\pi = (d_1, \dots, d_n)$ is potentially $K_4 - K_2 \cup K_2$ -graphic if it has a realization containing an $K_4 - K_2 \cup K_2$ as a subgraph where K_4 is a complete graph on four vertices and $K_2 \cup K_2$ is a set of independent edges. In this paper, we find the smallest degree sum such that every *n*-term graphical sequence contains $K_4 - K_2 \cup K_2$ as subgraph.

Key Words: Graph, (*a*,*b*,*n*)-graph, potentially graphical sequences.

AMS Subject Classifications: 05C07

1 Introduction

Let G(V, E) be a simple graph (a graph without multiple edges and loops) with n vertices and m edges having vertex set $V(G) = \{v_1, v_2, \dots, v_n\}$. The set of all non-increasing nonnegative integer sequences $\pi = (d_1, d_2, \dots, d_n)$ is denoted by NS_n . A sequence $\pi \in NS_n$ is said to be graphic if it is the degree sequence of a simple graph G on n vertices, and such a graph G is called a realization of π . The set of all graphic sequences in NS_n is denoted by GS_n . There are several famous results, Havel and Hakimi [7,8] and Erdös and Gallai [2] which give necessary and sufficient conditions for a sequence $\pi = (d_1, d_2, \dots, d_n)$ to be the degree sequence of a simple graph G. Another characterization of graphical sequences can be seen in Pirzada and Yin Jian Hu [15]. A graphical sequence π is potentially Hgraphical if there is a realization of π containing H as a subgraph, while π is forcibly Hgraphical if every realization of π contains H as a subgraph. A sequence $\pi = (d_1, d_2, \dots, d_n)$

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