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Some fixed point theorems for single valued and set valued maps

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Abstract. In this paper we introduce the concept of single and set valued maps and obtain new types of contraction mappings both for single valued map and set valued maps and establish new fixed point theorems in this direction.

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1. Introduction

The Banach contraction principle (fixed point theorem) which appeared in the Banach's thesis in 1922 [2] where it was used to establish the existence of a solution to an integral equation. In [17], Nadler extended the Banachs fixed point theorem from single valued map to set valued map. The other fixed point theorems for set valued mappings can be seen in [8,13,21,23,24,26]. The Banach's contraction principle [2] is the first principle on the fixed points for contractive type mappings. Because of its simplicity number of authors have obtained many interesting extensions and generalizations of the Banachs contractions principle [3,5–7,15,25] and references there in. Some of such generalizations are obtained by contraction conditions described by rational expressions [10,18,19]. Through out this paper, X is assumed to be the non empty set. The concepts of K-contraction and C-contraction have been introduced respectively by Kannan [12] and Chatterjea [4] as follows.

Definition 1.1. *Let (X, d) be a metric space. A mapping $T : X \to X$ is said to be*

(i) *a C-contraction [4] if there exists $\alpha \in \left[0, \frac{1}{2}\right)$ such that*

$$d(Tx, Ty) \le \alpha[d(x, Ty) + d(y, Tx)]$$

for all $x, y \in X$.

(ii) *a K-contraction [12] if there exists $\alpha \in \left[0, \frac{1}{2}\right)$ such that*

$$d(Tx, Ty) \le \alpha[d(x, Tx) + d(y, Ty)]$$

for all $x, y \in X$.

(iii) *a Reich contraction [20] if and only if for all $x, y \in X$, there exists non-negative numbers q, r, and s such that $q + r + s < 1$ and*

$$d(Tx, Ty) \le qd(x, y) + rd(x, Tx) + sd(y, Ty).$$

(iv) *a Ciric contraction [5] if and only if for all $x, y \in X$, there exists non-negative numbers q, r, s and t such that $q + r + s + 2t < 1$ and*

$$d(Tx, Ty) \le qd(x, y) + rd(x, Tx) + sd(y, Ty) + t[d(x, T(y)) + d(y, T(x))].$$