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Bounds for the energy of weighted graphs

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ABSTRACT

Let G be a simple connected graph with n vertices and m edges. Let $W(G) = (G, w)$ be the weighted graph corresponding to G. Let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be the eigenvalues of the adjacency matrix $A(W(G))$ of the weighted graph $W(G)$. The energy $\mathbb{E}(W(G))$ of a weighted graph $W(G)$ is defined as the sum of absolute value of the eigenvalues of $W(G)$. In this paper, we obtain upper bounds for the energy $\mathbb{E}(W(G))$, in terms of the sum of the squares of weights of the edges, the maximum weight, the maximum degree d_1, the second maximum degree d_2 and the vertex covering number τ of the underlying graph G. As applications to these upper bounds we obtain some upper bounds for the energy (adjacency energy), the extended graph energy, the Randić energy and the signed energy of the connected graph G. We also obtain some new families of weighted graphs where the energy increases with increase in weights of the edges.

1. Introduction

Let G be simple graph with n vertices and m edges having vertex set $V = V(G) = \{v_1, v_2, \ldots, v_n\}$ and edge set $E = E(G) = \{e_1, e_2, \ldots, e_m\}$. Throughout this paper by a graph G, we mean the graph $G = (V, E)$ having n vertices and m edges, unless otherwise stated. The adjacency matrix $A = (a_{ij})$ of G is a $(0, 1)$-square matrix of order n whose (i, j)th-entry is equal to 1, if v_i is adjacent to v_j and equal to 0, otherwise. The spectrum of the adjacency matrix is called the adjacency spectrum of the graph G.

Let $x_1, x_2, \ldots, x_n$ be the adjacency eigenvalues of G. The energy [15] of G is defined as

$$\mathbb{E}(G) = \sum_{i=1}^{n} |x_i|.$$

This concept was introduced by Gutman and is intensively studied in chemistry, since it can be used to approximate the total π-electron energy of a molecule see [20] and the references therein. This spectrum-based graph invariant has been much studied in both chemical and mathematical literature. Among the pioneering results of the theory of graph energy are the lower and upper bounds for energy see [1,20,26] and the references therein. For some more results on energy and some new results involving other concepts of related energies, we refer to [7–12,16,23–25].

An (edge)-weighted graph $W(G)$ is defined to be an ordered pair (G, w) where $G = (V, E)$ is the underlying graph of $W(G)$ and $w : E \to \mathbb{R}$ is the weight function, which assigns to each edge $e \in E(G)$ a non-zero weight $w(e)$. Every graph can be regarded as the weighted graph with weight of each edge equal to one. Thus weighted graphs are generalizations of graphs.

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