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Bounds for the skew Laplacian spectral radius of oriented graphs

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ABSTRACT. We consider the skew Laplacian matrix of a digraph $\overrightarrow{G}$ obtained by giving an arbitrary direction to the edges of a graph G having n vertices and m edges. We obtain an upper bound for the skew Laplacian spectral radius in terms of the adjacency and the signless Laplacian spectral radius of the underlying graph G. We also obtain upper bounds for the skew Laplacian spectral radius and skew spectral radius, in terms of various parameters associated with the structure of the digraph $\overrightarrow{G}$ and characterize the extremal graphs.

1. INTRODUCTION

Consider a simple graph G with n vertices and m edges and having the vertex set $V = \{v_1, v_2, \dots, v_n\}$. Let $\overrightarrow{G}$ be a digraph obtained by assigning arbitrarily a direction to each of the edges of G. The digraph $\overrightarrow{G}$ is called an orientation of G or oriented graph corresponding to G. Also the graph G is called the underlying graph of $\overrightarrow{G}$. Let $d_i^+ = d^+(v_i)$, $d_i^- = d^-(v_i)$ and $d_i = d_i^+ + d_i^-$, $i = 1, 2, \dots, n$ be respectively the out-degree, in-degree and degree of the vertices of $\overrightarrow{G}$. The out-adjacency matrix of the digraph $\overrightarrow{G}$ is the $n \times n$ matrix $A^+ = A^+(\overrightarrow{G}) = (a_{ij})$, where $a_{ij} = 1$, if (v_i, v_j) is an arc and $a_{ij} = 0$, otherwise. The in-adjacency matrix of the digraph $\overrightarrow{G}$ is the $n \times n$ matrix $A^- = A^-(\overrightarrow{G}) = (a_{ij})$, where $a_{ij} = 1$, if (v_j, v_i) is an arc and $a_{ij} = 0$, otherwise. We note that $A^- = (A^+)^t$. The skew adjacency matrix of a digraph $\overrightarrow{G}$ is the $n \times n$ matrix $S = S(\overrightarrow{G}) = (s_{ij})$, where

$$s_{ij} = \begin{cases} 1, & \text{if there is an arc from } v_i \text{ to } v_j, \\ -1, & \text{if there is an arc from } v_j \text{ to } v_i, \\ 0, & \text{otherwise.} \end{cases}$$

Clearly $S(\overrightarrow{G})$ is a skew symmetric matrix, so all its eigenvalues are zero or purely imaginary. For recent developments on the theory of skew spectrum, we refer to the papers [2, 14, 16, 23, 26, 28]. Let $D^+ = D^+(\overrightarrow{G}) = diag(d_1^+, d_2^+, \dots, d_n^+)$, $D^- = D^-(\overrightarrow{G}) = diag(d_1^-, d_2^-, \dots, d_n^-)$ and $D(\overrightarrow{G}) = diag(d_1, d_2, \dots, d_n)$ be respectively the diagonal matrix of vertex out-degrees, vertex in-degrees and vertex degrees of $\overrightarrow{G}$. Further, let A^+ and A^- be respectively the out-adjacency and in-adjacency matrix of a digraph $\overrightarrow{G}$. If $S(\overrightarrow{G})$ is the skew adjacency matrix of $\overrightarrow{G}$ and $A(G)$ is the adjacency matrix of the underlying graph G of the digraph $\overrightarrow{G}$, clearly $A(G) = A^+ + A^-$ and $S(\overrightarrow{G}) = A^+ - A^-$. Analogous to the definition of Laplacian matrix of a graph, Cai et al. [4] called the matrix $\widetilde{SL}(\overrightarrow{G}) = \widetilde{D}(\overrightarrow{G}) - S(\overrightarrow{G})$, where $\widetilde{D}(\overrightarrow{G}) = D^+(\overrightarrow{G}) - D^-(\overrightarrow{G})$, as the *skew Laplacian matrix* of the digraph

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