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THE CONDITION FOR A SEQUENCE TO BE POTENTIALLY $A_{L,M}$- GRAPHIC

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Abstract. The set of all non-increasing non-negative integer sequences $\pi = (d_1, d_2, \ldots, d_n)$ is denoted by NS_n. A sequence $\pi \in NS_n$ is said to be graphic if it is the degree sequence of a simple graph G on n vertices, and such a graph G is called a realization of π. The set of all graphic sequences in NS_n is denoted by GS_n. The complete product split graph on $L+M$ vertices is denoted by $\overline{S}_{L,M} = K_L \vee \overline{K}_M$, where K_L and K_M are complete graphs respectively on $L = \sum_{i=1}^{p} r_i$ and $M = \sum_{i=1}^{p} s_i$ vertices with r_i and s_i being integers. Another split graph is denoted by $S_{L,M} = \overline{S}_{r_1,s_1} \vee \overline{S}_{r_2,s_2} \vee \cdots \vee \overline{S}_{r_p,s_p} = (K_{r_1} \vee \overline{K}_{s_1}) \vee (K_{r_2} \vee \overline{K}_{s_2}) \vee \cdots \vee (K_{r_p} \vee \overline{K}_{s_p})$. A sequence $\pi = (d_1, d_2, \ldots, d_n)$ is said to be potentially $S_{L,M}$-graphic (respectively $\overline{S}_{L,M}$)-graphic if there is a realization G of π containing $S_{L,M}$ (respectively $\overline{S}_{L,M}$) as a subgraph. If π has a realization G containing $S_{L,M}$ on those vertices having degrees $d_1, d_2, \ldots, d_{L+M}$, then π is potentially $A_{L,M}$-graphic. A non-increasing sequence of non-negative integers $\pi = (d_1, d_2, \ldots, d_n)$ is potentially $A_{L,M}$-graphic if and only if it is potentially $S_{L,M}$-graphic. In this paper, we obtain the sufficient condition for a graphic sequence to be potentially $A_{L,M}$-graphic and this result is a generalization of that given by J. H. Yin on split graphs.

1. Introduction

Let $G(V, E)$ be a simple graph (a graph without multiple edges and loops) with n vertices and m edges having vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$. The set of all non-increasing non-negative integer sequences $\pi = (d_1, d_2, \ldots, d_n)$ is denoted by NS_n. A sequence $\pi \in NS_n$ is said to be graphic if it is the degree sequence of a simple graph G on n vertices, and such a graph G is called a realization of π. The set of all graphic sequences in NS_n is denoted by GS_n. There are several famous results, Havel and Hakimi [3, 4] and Erdös and Gallai [1] which give necessary and sufficient conditions for a sequence

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