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Nonlinear dispersion relation of dust acoustic waves using the Korteweg–de Vries model

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ABSTRACT

In this Brief Communication, we present an exact analytic nonlinear dispersion relation (NLDR) for the dust acoustic waves using the Korteweg–de Vries model. The NLDR agrees with the spectrum of spatiotemporal evolution obtained from an exact solution as in Mir *et al.* [Phys. Plasmas **27**, 113701 (2020)]. The NLDR also shows a reasonable match with the experimental data of Thompson *et al.* [Phys. Plasmas **4**, 2331 (1997)] in the long-wavelength limit ($k\lambda_D \ll 1$). We suggest that such nonlinear corrections should be incorporated in the dispersion relation along with damping, streaming, and correlation effects in order to provide a more realistic interpretation of experimental data.

Dust acoustic waves (DAWs) are analogs of ion-acoustic waves in a dusty plasma that exist due to the balance of charged dust inertia and plasma pressure.[1,2] These waves can arise naturally[3–7] or can be excited by an external perturbation.[8] They are low-frequency waves in the range of a few tens of Hz with wavelengths of a few tens of mm. Their characteristics slow timescales are due to the heavy mass of the micrometer-sized dust particles. The experimentally observed space–time evolution of DAWs usually shows nonlinear features through the appearance of harmonics in the frequency and wave number domains.[9–11] The experimental data are frequently compared with theoretically derived linear dispersion relations that customarily incorporate various linear contributions arising from collisional/kinetic damping, particle streaming, and correlation effects. However, since experimental data are based on measurements of finite amplitude waves, its nature cannot be fully captured in a linear model-based dispersion relation. In this paper, we propose a nonlinear dispersion relation (NLDR) that is based on the Korteweg–de Vries (KdV) model and that can provide a better description of the DAW data in the weakly nonlinear dynamical regime.

DAWs show a sound wave nature in the long-wavelength limit,[4,5] a dispersive nature for short wavelengths, and a phase velocity reversal in the strongly coupled regime.[12] As one of the fundamental modes of a dusty plasma, it has been an object of intense research ever since its theoretical prediction by Rao *et al.*[1] and its experimental identification thereafter by Barkan *et al.*[13] Its linear properties were thoroughly explored in weak and strong coupling regimes using fluid models,[14–16] quasi-localized charge approximation,[17,18] molecular dynamics simulations,[18,19] and in laboratory experiments.[13]

A simple linear dispersion for DAW, $\omega/k \approx \lambda_D\omega_{pd} = C_{pd}$, was proposed by Rao *et al.*[1] in the long-wavelength limit $k\lambda_D \ll 1$. Here, C_{pd} is the dust sound speed in the medium, λ_D is the Debye length due to Boltzmann ions and electrons, and ω_{pd} is the characteristic frequency of DAW. For experimental conditions, effects due to dust-neutral collisions, ion streaming, and other damping mechanisms can be significant. Ruhunusiri *et al.*[20] have extensively explored the dispersion relation of DAW by retaining most of the above-mentioned effects except nonlinear corrections arising from the finite size of the wave amplitude. In subsequent work, Goree *et al.*[21] provided a kinetic dispersion relation for DAWs and compared it with data from the Plasma Kristall-4 (PK-4) experiment on the International Space Station. Interestingly, we found that these experimental data also show features arising from nonlinearity of the propagating wave. However, such nonlinear effects have not been incorporated so far in the context of a dispersion relation.

The KdV equation successfully models low-frequency weakly nonlinear wave propagation in a variety of media including dusty plasmas.[22,23] The conoidal waves and solitons are exact solutions of the KdV equation and have found applications in domains like hydrodynamics,[24] oceanography,[25] plasmas,[2] nonlinear optics,[26] and astrophysical systems.[27] Optical fiber communication is one such practical