

# Modeling of experimentally observed two-dimensional precursor solitons in a dusty plasma by the forced Kadomtsev-Petviashvili equation

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We compare model solutions of a forced Kadomtsev-Petviashvili (fKP) equation with experimental observations of dust acoustic precursor solitons excited by a supersonically moving charged cylindrical object in a dusty plasma medium. The fKP equation is derived from a three-fluid-Poisson model of the dusty plasma using the reductive perturbation technique and numerically solved for parameters close to the experimental investigations of cylindrical precursor solitons. The fKP model solutions show excellent agreement with the experimental results in reproducing the prominent geometric features of the two-dimensional solitons and closely matching the quantitative values of their velocities, amplitudes, and temporal evolutions. Our findings suggest that the fKP equation can serve as a very realistic model to investigate the dynamics of precursor solitons and can be usefully employed in practical applications such as space debris detection and tracking techniques that are based on observing/predicting nonlinear plasma excitations induced by the debris in the ionosphere.

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## I. INTRODUCTION

The topic of precursor and pinned solitons excited by a fast-moving charged object in a plasma has received much attention lately after it was first suggested in [1] that such solitons could potentially prove useful in the detection and tracking of small-sized space debris objects orbiting the Earth in the low Earth orbital (LEO) and geosynchronous Earth orbital (GEO) regions [2–8]. These objects, which are difficult to detect optically and possess enormous kinetic energy, can cause serious damage to active space assets through their collisional impacts. Their detection and tracking are therefore of paramount importance and continue to be a major area of research in the Space Situational Awareness program. One characteristic feature of space debris objects is that they get highly charged through the collection of electrons and ions from the surrounding plasma and a variety of other processes like the production of photo-electrons due to ultraviolet and x-rays from the sun and the emission of secondary electrons and ions from the surface [1,9,10]. As charged objects, they can then interact electromagnetically with the plasma to create a variety of linear and nonlinear waves. Under certain conditions, dependent on the speed and amount of charge on the object, they can give rise to coherent nonlinear structures like precursor solitons. Precursor solitons are nonlinear localized wave structures that travel faster than the source exciting them and propagate for a long distance. They satisfy

the mathematical condition of the constancy of the quantity  $A_p W_p^2$ , where  $A_p$  is the amplitude and  $W_p$  is the width of the structure. Precursor solitons have been extensively studied in the context of the one-dimensional forced Korteweg-de Vries (fKdV) equation to model such excitations in hydrodynamics as well as in plasmas [1,2,11–13]. The precursor solitons of the forced Kadomtsev-Petviashvili (fKP) equation, discussed in the present work, are the two-dimensional generalization of the fKdV solitons that possess a transverse structure that gives them a cylindrical or spherical shell shape. The detection and tracking of such nonlinear plasma signatures, which can have a larger footprint than the size of the object, formed the basis for the technique suggested in [1]. It was further shown that a simple one-dimensional (1D) model equation in the form of the fKdV equation was capable of capturing the basic features of the excitation and propagation of precursor solitons in the plasma and could be usefully employed to develop this scheme further. Subsequently, the fKdV equation was extensively investigated by Truitt *et al.* [2,3] to explore the feasibility of the scheme for ionospheric conditions. To account for higher-dimensional effects, a two-dimensional (2D) generalization of the fKdV equation in the form of the fKP equation was also numerically investigated by Truitt *et al.* [4] in different parameter regimes.

While the fKdV and fKP serve as excellent mathematical models for a qualitative understanding of the basic features of precursor soliton excitation and propagation characteristics, it is necessary to validate their applicability in practical situations by comparing their solutions with experimental results. This is the primary motivation of the present paper, where we test the validity of the fKP equation in quantitatively

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