Synchronization of dust acoustic waves in a forced Korteweg–de Vries–Burgers model

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The synchronization of dust acoustic waves to an external periodic source is studied in the framework of a driven Korteweg–de Vries–Burgers equation that takes into account the appropriate nonlinear and dispersive nature of low-frequency waves in a dusty plasma medium. For a spatiotemporally varying source term, the system is shown to demonstrate harmonic (1:1) and superharmonic (1:2) synchronized states. The existence domains of these states are delineated in the form of Arnold tongue diagrams in the parametric space of the forcing amplitude and forcing frequency and their resemblance to some past experimental results is discussed.

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I. INTRODUCTION

The nonlinear phenomenon of frequency synchronization is ubiquitous in many physical, chemical, and biological systems and has been the subject of a large number of studies over the past several years [1–3]. The simplest mathematical model describing this phenomenon consists of an ensemble of globally coupled nonlinear point oscillators that adjust their intrinsic frequencies to a common collective frequency as the coupling strength is increased [4–7]. Such a nonlinear phenomenon can also be observed in a continuum medium (a fluid) where a self-excited oscillation or a wave can interact with a driving force and adjust its oscillation or wave frequency [8–13]. A plasma system with its wide variety of collective modes and complex nonlinear dynamics provides a rich and challenging medium for the exploration of synchronization phenomena. A number of past experimental studies have examined the driven response of a plasma to an external frequency source [9–11,14–23]. These studies include the synchronization of waves and oscillations at ion and dust dynamical scales as well as chaos and wave turbulence. There have also been a few studies devoted to an investigation of mutual synchronization between two plasma devices [24–26].

More recently, synchronization phenomena have been experimentally explored in dusty plasma devices where it is easy to visualize the low-frequency wave activity using fast video imaging. A dusty plasma is a four-component plasma of electrons, ions, neutral gas atoms, and micron-size particles of solid matter [27–29]. It can be produced in a laboratory device like a glow discharge plasma, by introducing micron-sized solid particles [30–33]. These small solid particles (dust) get negatively charged by absorbing more electrons which have a higher mobility than ions. Such a charged medium consisting of dust, ions, and electrons can sustain a variety of collective modes [29,34–36]. The dust acoustic wave (DAW) or dust density wave (DDW) first theoretically predicted by Rao *et al.* [37] is one such well-known low-frequency compressional mode that is analogous to the ion acoustic wave [29,38]. A DAW can be spontaneously excited due to the onset of an ion-streaming instability. The DAW has a very low frequency (typically 10–100 Hz) [14,30] due to the large mass of the dust particles and can consequently be visually observed through its images and video recording [31,39–41]. The term "dust density wave" originated as a generalization of "dust acoustic wave," after observing wavefronts (visible in the dust cloud) that appeared to be oblique with respect to the ion drift direction [42]. Two key factors led to the use of the term DDW, namely, the presence of ion drift and an oblique orientation of the wavefront and its propagation with respect to the ion drift. Since then, many research groups have used the term "dust density wave" and "dust acoustic wave" synonymously [14,31,35,43–45]. The present work focuses on the synchronization of DAW using the forced Korteweg–de Vries–Burgers (fKdV-B) model.

Synchronization of dust acoustic waves has been studied in anodic [15], radio-frequency (RF), and direct-current (DC) plasmas [14,16,46]. Pilch *et al.* [15] reported the entrainment of DAWs through a driving modulation to the anode. Ruhunusiri *et al.* [14] reported observation of harmonic, superharmonic, and subharmonic synchrony of self-excited cnoidal DAWs. This was achieved through the driven modulation of the streaming ions in the dust cloud. Their experiments showed parametric regions for the occurrence of such synchrony in the form of Arnold tongue diagrams in the state space of the driving frequency and driving amplitude. They also observed features like the branching of the tongues and the existence of an amplitude threshold for synchronization to occur. Williams *et al.* [16] compared DAW synchronization in RF and DC generated plasmas. Their results suggested that in a RF plasma, synchronization was restricted to a part

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