Lindley Approximation Technique for the Parameters of Lomax Distribution

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Abstract: The present study is concerned with the estimation of shape and scale parameter of Lomax distribution using Bayesian approximation techniques (Lindley's Approximation). Different priors viz gamma, exponential and Levy priors are used to obtain the Bayes estimates of parameters of Lomax distributions under Lindley approximation technique. For comparing the efficiency of the obtained results a simulation study is carried out using R-software.

Keywords: Lomax distribution, Bayesian Estimation, Prior, Loss functions, Lindley's Approximation.

1. INTRODUCTION

The Lomax distribution also known as Pareto distribution of second kind has, in recent years, assumed opposition of importance in the field of life testing because of its uses to fit business failure data. It has been used in the analysis of income data, and business failure data. It may describe the lifetime of a decreasing failure rate component as a heavy tailed alternative to the exponential distribution. Lomax distribution was introduced by Lomax (1954), Abdullah and Abdullah (2010) [1, 2] estimated the parameters of Lomax distribution based on generalized probability weighted moment. Zangan (1999) [3] deals with the properties of the Lomax distribution with three parameters. Abd-Elfatth and Mandouh (2004) [4] discussed inference for R = Pr{Y<X} when X and Y are two independent Lomax random variables. Afaq *et al*. (2015) [5] performs comparisons of maximum likelihood estimation (MLE) and Bayes estimates of shape parameter using prior distribution. Afaq *et al*. [6] proposed Length biased Weighted Lomax distribution and discussed its structural properties. The cumulative distribution function of Lomax distribution is given by

$$F\left(x;\alpha,\beta\right)=1-(1+\beta x)^{-\alpha}\,,\quad x>0,\ \ \alpha,\beta>0, \tag{1.1}$$

and the corresponding probability density function is given by

$$f\left(x;\alpha,\beta\right)=\alpha\beta(1+\beta x)^{-(\alpha+1)},\ \ x>0,\ \ \alpha,\beta>0, \tag{1.2}$$

where α and β are the shape and scale parameters respectively.

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The Lomax distribution has not been discussed in detail under the Bayesian approach. The Bayesian paradigm is conceptually simple and probabilistically elegant. Sometimes posterior distribution is expressible in terms of complicated analytical function and requires intensive calculation because of its numerical implementations. It is therefore useful to study approximate and large sample behavior of posterior distribution. Uzma (2017) [7] obtains the estimate of shape parameter of inverse Lomax distribution. Our present study aims to obtain the estimate the shape and scale parameters of Lomax distribution using Lindley approximation technique using Gamma prior, Exponential prior and Inverse Levy prior. A simulation study has also been conducted along with concluding remarks.

2. LINDLEY APPROXIMATION

Sometimes, the integrals appearing in Bayesian estimation can't be reduced to closed form and it becomes tedious to evaluate of the posterior expectation for obtaining the Baye's estimators. Thus, we propose the use of Lindley's approximation method (1980) [8] for obtaining Baye's estimates. Lindley developed an asymptotic approximation to the ratio

$$I(X)=\frac{\int_{(\alpha,\beta)}U(\alpha,\beta)e^{L(\alpha,\beta)+\rho(\alpha,\beta)}\partial(\alpha,\beta)}{\int_{(\alpha,\beta)}e^{L(\alpha,\beta)+\rho(\alpha,\beta)}\partial(\alpha,\beta)},$$

where $U(\alpha,\beta)$ is function of α and β only and $L(\alpha,\beta)$ is the log-likelihood and $\rho(\alpha,\beta)=\log g\left(\alpha,\beta\right)$. Let $(\hat{\alpha},\hat{\beta})$ denotes the MLE of (α,β). For sufficiently large sample size n, using the approach developed by Lindley (1980) [8], the ratio of integral $I(X)$ as defined above can be written as

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