

A simple way of improving the Bar-Lev, Bobovitch and Boukai Randomized response model

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Abstract

Eichhorn & Hayre (1983) considered a randomized response procedure suitable for estimating the mean response, when the sensitive variable under investigation is quantitative in nature. They have obtained an estimate for the mean of the quantitative response variable under investigation and studied its properties. Bar-Lev *et al.* (2004) have suggested an alternative procedure, which use a design parameter (controlled by the experimenter) that generalizes Eichhorn & Hayre's (1983) results. They have also proved that the estimator proposed by them has uniformly smaller variance as compared to that of Eichhorn & Hayre (1983) in certain condition. In this paper we have suggested a simple procedure of improving the Eichhorn & Hayre (1983) and Bar-Lev *et al.* (2004) models along with its properties. It has been shown that the proposed procedure is uniformly better than Bar-Lev *et al.* (2004) procedure. The proposed procedure is also uniformly better than Eichhorn and Hayre's (1983) procedure under the same condition in which the Bar-Lev *et al.*'s (2004) procedure is more efficient than Eichhorn & Hayre's (1983) procedure. Numerical illustration is given in support of the present study.

Keywords: Estimation of proportion; randomized response sampling; respondents protection; sensitive quantitative variable.

AMS Subject Classification: 62D05.

1. Introduction

The problem of estimating the population mean of a sensitive quantitative variable is well recognized in survey sampling. Randomized response techniques (RRT) have been extensively used for personal interview surveys, ever since the pioneering work of Warner (1965). A rich growth of literature can be found in Fox & Tracy (1986), Chaudhuri & Mukerje (1988), Singh (2003) and among others. For recent references readers are referred to Gjestvang & Singh (2006, 2009), Bar-Lev *et al.* (2004), Singh & Mathur (2004, 2005), Odumade & Singh (2008, 2009), Hussain (2012), Singh & Tarray (2013, 2016) and Tarray and Singh (2015, 2016, 2017). The present study rely on the models suggested by Eichhorn & Hayre (1983) and Bar-Lev *et al.* (2004) so the description of these models are respectively given in section 1.1 and 1.2.

1.1 Eichhorn & Hayre (1983) procedure:

Eichhorn & Hayre (1983) suggested a multiplicative model to collect information on sensitive quantitative variables like income, tax evasion and amount of drug used. By their procedure, the interviewees are asked about their value of sensitive response variable. In return, they are allowed to respond with a coded (or scrambled) value composed of their true value for the variable of interest,

multiplied by some random number. The interviewer does not know which random number was used by each of the interviewees for coding their responses, but fully knows the underling distribution which generated the coding number.

Let X be a random variable denoting the quantitative response variable of interest and let S be a random variable denoting the random number used in the coding mechanism. Suppose that $X (\geq 0)$ is independent of S and let $Y = SX$ the coded response returns to the interviewee to the sensitive question, see Bar-Lev *et al.* (2004, p. 256). It is assumed that the distribution of the scrambling variable S is known. In other words,

$$\mu_x = E(X), \mu_s = E(S), \sigma_x^2 = V(X), \gamma^2 = V(S)$$

where μ_s and γ^2 are known and μ_x and σ_x^2 are unknown. We also denote $C_x = \sigma_x / \mu_x$ and $C_s = \gamma / \mu_s$ for the coefficient of variation of X and of S , respectively. The mean and variance of $Y = XS$ are respectively given by

$$E(Y) = \mu_x \mu_s \quad (1)$$

and

$$V(Y) = \sigma_x^2 \mu_s^2 + \mu_x^2 (1 + C_x^2) \gamma^2. \quad (2)$$

Eichhorn & Hayre (1983) based on a random sample