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Approximate Snapshot-ensemble Generation for Basis Extraction in Proper Orthogonal Decomposition

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Abstract: This paper tries to address the problem of high computational burden posed by the basis extraction procedure in Proper Orthogonal Decomposition (POD). A proposal for generating approximate snapshots of the nonlinear system, to reduce this burden is presented. Simulation results are shown which demonstrate that a substantial reduction in computational time with no or insignificant deterioration in performance is easily obtained.

Keywords: Simulation, Nonlinear Dynamical Systems, Proper Orthogonal Decomposition, Model Order Reduction.

1. INTRODUCTION

POD is a powerful data analysis technique that is used to extract basis functions from large data sets. These basis functions are then used to characterise the sets. In Model order reduction (MOR), the data is obtained from physical-experiments or simulations of large systems and the extracted basis represents the space in which solutions of the system are expected to lie. Also known as Principal component analysis or Karhunen-Loève decomposition, POD finds applications in diverse fields of engineering and science, like signal analysis, data-compression and weather prediction.

MOR of dynamical systems using POD consists three steps – data generation, basis extraction and projection. In the data generation step, the large system is made to respond to varied inputs and its state vector is collected and stacked at periodic intervals in a matrix called the snapshot ensemble. The inputs used for data generation are the ones that the system is expected to encounter in its physical setting. The extent of this training process, and correspondingly the richness of the snapshot ensemble, is a matter of choice and varies with applications. The next step, basis extraction, is based on the assumption that the snapshot matrix contains important dominant 'directions' (also called empirical eigenfunctions) in which the system is expected to evolve, and hence a dominant low-dimensional subspace can be obtained to characterise the system. The third and final step, is to simply project the large scale system onto the low-dimensional subspace.

There are many questions that arise automatically out of the above-mentioned process of MOR; the choice of inputs, the adequateness of the snapshot matrix, the question of the high computational cost of the projected non-linearity etc. Efforts have been made to address these problems, and different modifications of POD have been developed as a result. An important question is the cost of the model reduction procedure to which little attention has been paid.

The primary objective of MOR is the reduction of online simulation time, which depends only on the complexity of the Reduced order model (ROM) obtained. However, it is also wanted that the procedure of obtaining the ROM, which is offline, does not impose a heavy computational burden. This is to ensure that the process of generating and simulating a ROM leads to overall savings in computational resources and cost. In POD, simulation of the large-nonlinear dynamical system for generating the snapshot ensemble is a computationally expensive procedure. These simulations are done for multiple inputs and sometimes with varied system parameters to generate a ROM which is valid over a range of input and parameter variations. In this regard, it is evident that POD suffers from an important drawback – it requires computationally heavy, a-priori simulations. Although considered to be a matter of concern, no studies addressing this problem could be found in literature. The objective in this paper is to propose a new method of the snapshot-ensemble generation with a reduced computational demand, and obtain savings in the overall cost of the POD procedure.

The organisation of this paper is as follows. In the next section, POD is briefly reviewed. This is followed by a discussion on the peculiarities of numerically solving large systems governed by nonlinear differential equations. Subsequently, the new technique is proposed and validated on a benchmark Micro-electro-mechanical (MEMS) model.

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