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A scheme for comprehensive computational cost reduction in proper orthogonal decomposition

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This paper addresses the issue of offline and online computational cost reduction of the proper orthogonal decomposition (POD) which is a popular nonlinear model order reduction (MOR) technique. Online computational cost is reduced by using the discrete empirical interpolation method (DEIM), which reduces the complexity of evaluating the nonlinear term of the reduced model to a cost proportional to the number of reduced variables obtained by POD: this is the POD-DEIM approach. Offline computational cost is reduced by generating an approximate snapshot-ensemble of the nonlinear dynamical system, consequently, completely avoiding the need to simulate the full-order system. Two snapshot ensembles: one of the states and the other of the nonlinear function are obtained by simulating the successive linearization of the original nonlinear system. The proposed technique is applied to two benchmark large-scale nonlinear dynamical systems and clearly demonstrates comprehensive savings in computational cost and time with insignificant or no deterioration in performance.

K e y w o r d s: proper orthogonal decomposition, reduced order model, discrete empirical interpolation, approximate snapshot ensemble

1 Introduction

The aim of POD is to obtain a compact system by projecting a high-dimensional system into a low-dimensional subspace while retaining the dominant features of state evolution dynamics. The low dimensional subspace is obtained from state snapshots in response to certain inputs to which the high-dimensional system, ie a full order model (FOM) is subjected [1]. However, it is well known that this conventional method of generating a reduced-order model has an associated high computational cost related to the projected high-dimensional nonlinear function. The computational difficulties associated with nonlinear function were recognized early by the model order reduction (MOR) community, and a number of approaches were proposed to overcome these difficulties. These approaches include the missing point estimation (MPE) [2], "best points" method, empirical interpolation method (EIM) [3], and the gappy POD method [4–7]. The MPE method computes the Galerkin projection over a restricted subset of the spatial domain; the gappy POD is a used in the case of sparse measurements. The applications of these methods are closely related to the number of spatial grid points. For a small number of grid points, MPE fails to converge whereas, for the same small number of grid points, gappy POD, EIM and DEIM may converge [8]. EIM was proposed to avoid the complete evaluation of the nonlinear function and in the Jacobian matrix to work iteratively, whereas DEIM [9] is a discrete variant of EIM and represents a well-received effort towards a solution of the nonlinear function.

In [10], which is related to this work, attention of the MOR community has been drawn to the offline computational cost of the POD. In [11], a method was suggested to reduce the procedural-cost for obtaining the reduced-order model, which is offline, so that the process of generating and simulating a reduced-order model leads to overall savings in computational resources. This was done by eliminating the need for computationally heavy, prior simulations of the high-dimensional problem to generate state snapshot-ensembles by proposing to generate an approximate snapshot ensemble obtained from simulations of successive linearization of the nonlinear system.

This paper is an effort to extend the idea of approximate snapshots to the POD-DEIM approach. The aim is to reduce both the online and offline computational cost of the POD procedure. This is done by creating and using approximate snapshot ensembles for the states as well as the nonlinear term of the original high-dimensional system.

We review POD and DEIM, followed by sections on the generation of approximate snapshot ensembles and their subsequent application in DEIM for creating the reduced-order model. Calculation of computational savings are presented and, the proposed approach is validated on two benchmark models, showing improved offline and online computational performance with little to no loss of accuracy. Pseudo-codes are reproduced wherever necessary.

2 Review

2.1 *POD*

This method produces a low-order approximate description of a higher-order system. For an input func-

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