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AN INVENTIVE METHOD FOR ESTIMATING RARE AND SENSITIVE VARIABLES USING A RANDOMIZED RESPONSE METHODOLOGY†

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Abstract. This comprehensive study aims to delve deep into the process of determining the average quantity of individuals possessing a rare and delicate attribute through the utilization of stratified random sampling in conjunction with the widely acknowledged and widely used Poisson distribution. The inherent characteristics and properties of the recommended estimation procedures are meticulously examined, leaving no stone unturned in the pursuit of a thorough understanding. In order to provide further validation and substantiation to the theoretical findings, empirical studies have been conducted, employing real-world data to support the claims made. The results of these empirical investigations overwhelmingly demonstrate the superiority and pre-eminence of the proposed estimators in comparison to the existing estimators that have gained recognition and acceptance within the field. By carefully analyzing and interpreting the outcomes, valuable insights are obtained, resulting in the formulation and presentation of pertinent recommendations that are specifically targeted towards practitioners in the survey domain, thereby enabling them to enhance and refine their methodologies.

AMS Mathematics Subject Classification : 62D05. *Key words and phrases* : Estimation stratified random sampling, Poisson distribution, empirical studies, attribute prevalence.

1. Introduction

In a world that is characterized by an ever-expanding reservoir of data and an unyielding thirst for knowledge, the ability to accurately quantify the presence of rare and delicate attributes within a given population takes on a paramount

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level of importance. This research embarks on a journey that seeks to unravel the intricacies of this challenging endeavour, with the ultimate goal of shedding light on the elusive average quantity of individuals possessing such attributes. Through a fusion of methodological precision and empirical rigor, we aim to equip practitioners in the survey domain with robust tools and invaluable insights that will enhance their methodologies and improve their understanding of this complex phenomenon. Our pursuit commences with the strategic utilization of stratified random sampling, which is a well-established technique that is renowned for its capacity to produce samples that faithfully mirror the inherent diversity present in complex populations. By employing this methodological cornerstone in conjunction with the support of the Poisson distribution, a venerable statistical tool recognized for its aptitude in modelling rare events, we embark on a systematic exploration of attribute estimation. This harmonization of two pillars of statistical inquiry provides a solid foundation upon which we can build a comprehensive understanding of the estimation process. Studies in the field of socioeconomic research often require a deep exploration of personal attributes that individuals are inclined to keep concealed during extensive investigations. To gain a comprehensive understanding of the subject matter, researchers frequently deploy extensive questionnaires replete with a myriad of inquiries. While much of this information can be readily obtained through direct questioning, there exists a subset of inquiries that traverse sensitive terrain, prompting individuals to hesitate before offering forthright responses. These specific inquiries touch upon topics of a delicate nature, leading individuals to exercise caution when divulging information. For instance, individuals are often reticent to reveal accurate details regarding their financial reserves, the true extent of their wealth, involvement in intentional tax evasion, and engagement in other clandestine or unethical practices that yield income from illicit sources or criminal activities. Furthermore, inquiries may extend to matters such as involvement in the trade of illegal goods, inclinations towards substance abuse, and expenditures on various addictions, non-heterosexual orientations, and other facets of human life that society, in general, regards with disapproval. In cases such as these, straightforward or direct questions frequently fall short in eliciting reliable data concerning these confidential dimensions of human existence. Warner [1] developed an alternative survey technique that is known as randomized response (RR) technique. Later various modifications have been given by several researchers see Kim and Warde[2], Mangat and Singh[3]. Kim and Warde[2] have presented mixed randomized response models using simple random sampling with replacement sampling scheme which improves the privacy of respondents. In situations where potentially embarrassing or incriminating response are sought, the randomized response (RR) technique is effective in reducing non – sampling errors in sample surveys. Refusal to respond and lying in surveys are two main sources of such non – sampling errors, as the stigma attached to certain practices (e.g., sexual behaviours and the use of illegal drug)

oftentimes leads to discrimination. Warner [1] was first to introduce a randomized response Technique (RRT) model to estimate the proportion for sensitive attributes including homosexuality, drug addiction or abortion. Since the work by Warner [1], a huge literature has emerged on the use and formulation of different randomization device to estimate the population proportion of a sensitive attribute in survey sampling. Mention may be made of the work of Mangat and Singh[3], Naik P.A. [4,5],Singh and Tarray [6,7], Tabbasum et. al. [8], Tarray et al. [9,10,11] , and Tarray and Ganie [12,13] and the references cited therein.Singh et al. [14] used the randomization device carrying three types of cards bearing statements:
(i) "I belong to sensitive group A_1,"
(ii) " I belong to group A_2," and
(iii) " Blank cards," with corresponding probabilities Q_1,Q_2, and Q_3, respectively, such that $\sum_{i=1}^{3} Q_i = 1$. In case the blank card is drawn by the respondent, he/she will report "no." The rest of the procedure remains as usual. The probability of "yes" answer is given by

$$\theta_1 = Q_1\pi_1 + Q_2\pi_2 \tag{1}$$

where π_1 and π_2 are the true proportion of the rare sensitive attribute A_1 and the rare unrelated attribute A_2 in the population, respectively. From the above Equation (1) the estimator of π_1 is as

$$\hat{\pi_1} = \frac{\hat{\theta_1} + P_2\pi_2}{P_1} \tag{2}$$

where $\hat{\theta_1}$ the observed proportion of $\backslash yes$" answers in the sample. The variance of the estimator is $\hat{\pi_1}$ given as

$$V(\hat{\pi_1}) = \frac{\pi_1(1-\pi_1)}{n} + \frac{\pi_1(1 - P_1\pi_1 - 2P_2\pi_2)}{nP_1} + \frac{P_2\pi_2(1 - P_2\pi_2)}{nP_1^2} \tag{3}$$

Land et al. [15] suggested a method for estimating the mean number of individuals with a rare sensitive attribute by using the Poisson distribution. Motivated by Lee et al. [16] we scrutinize the inherent characteristics and properties of the recommended estimation procedures with a keen eye for detail, leaving no stone unturned. This meticulous approach forms the bedrock of our quest for a profound understanding of the estimation process. Our journey does not remain confined to the theoretical realm. In a bid to breathe life into our findings, we embark on empirical studies that leverage real-world data. These empirical investigations serve as a crucible, testing the efficacy and validity of our proposed estimators. It is through the crucible of empirical scrutiny that the strength and superiority of our methodology come to the fore, overshadowing existing estimators that have hitherto held sway within the field.

2. Proposed models : Sensitivity Estimation with Known Attributes

Let Ω be a finite population of size N which is composed into L strata of size N_h $(h = 1, 2, 3, ..., L)$. Sample of size n_h is drawn by simple random sampling with replacement (SRSWR) from h^{th} stratum. It is assumed that π_{h2} is known. The n_h respondents from h^{th} stratum at provided with following three stage randomization device: First-stage randomization device R_{1h} consists of two statements

Probability	*Statements*	*Selection*
Staement 1	*Are you a memeber of a rare sensitive Group* A_1?	T_h
Staement 2	*Go to randomization device* R_{2h}	$(1 - T_h)$

Second-stage randomization device R_{2h} consists of two statements

Probability	*Statements*	*Selection*
Staement 1	*Are you a memeber of a rare sensitive Group* A_1?	P_h
Staement 2	*Go to randomization device* R_{3h}	$(1 - P_h)$

The randomization device R_{3h} used three statements which are same as Singh et al. [14]. The probability of getting answer \yes" in hth stratum from the respondent using the above mentioned randomized response devices is

$$\theta_{h0} = T_h + (1 - T_h)\left[P_h\pi_{h1} + \frac{(1 - \pi_{h1})}{\pi_{h1}}(1 - P_h)\{Q_{1h}\pi_{h1} + Q_{2h}\pi_{h2}\}\right] \quad (4)$$

Where π_{h1} and π_{h2} are the true proportion of the rare sensitive attribute A_1 and the rare unrelated non-sensitive attribute A_2 in the population, respectively. Since, A_1 and A_2 are rare attributes, assuming that $n_h \to \infty$ and $\theta_{h0} \to 0$ such that $n_h\theta_{h0} = \lambda_{h0}$ is finite and follows Poisson distribution with parameter λ_{h0}.

$$\lambda_{h0} = T_h + (1 - T_h)\left[P_h\lambda_{h1} + \frac{(1 - \pi_{h1})}{\pi_{h1}}(1 - P_h)\{Q_{1h}\lambda_{h1} + Q_{2h}\lambda_{h2}\}\right] \quad (5)$$

The likelihood function based on the sample is defined as

$$L = \prod_{i=1}^{nh} \frac{e^{-\lambda h_0}\lambda_{h0}^{yhi}}{y_{hi}!}$$

The natural log-likelihood function is given by

$$\log L = -n_h\left[T_h + (1 - T_h)\left[P_h\lambda_{h1} + \frac{(1 - \pi_{h1})}{\pi_{h1}}(1 - P_h)\{Q_{1h}\lambda_{h1} + Q_{2h}\lambda_{h2}\}\right]\right]$$
$$+ \sum_{i=1}^{nh} y_{hi} \log\left[T_h + (1 - T_h)\left[P_h\lambda_{h1} + \frac{(1 - \pi_{h1})}{\pi_{h1}}(1 - P_h)\{Q_{1h}\lambda_{h1} + Q_{2h}\lambda_{h2}\}\right]\right]$$
$$- \sum_{i=1}^{nh} \log y_{hi} \quad (6)$$

Differentiating equation (6) with respect to λ_{h1} and equating to zero. We have the maximum likelihood estimator of λ_{h1} in the h^{th} stratum is given as

$$\begin{aligned}\hat{\lambda_{h1}} &= \frac{1}{T_h+(1-T_h)P_h+(1-T_h)\dfrac{(1-\pi_{h1})}{\pi_{h1}}(1-P_h)Q_{1h}} \\ &+ \left[\frac{1}{nh}\sum_{i=1}^{nh} y_{hi} - (1-T_h)\frac{(1-\pi_{h1})}{\pi_{h1}}(1-P_h)Q_{2h}\lambda_{h2}]\right]\end{aligned} \tag{7}$$

Therefore, the estimator $\hat{\lambda_1}$ for the mean number of persons in the population with rare sensitive characteristics $\hat{\lambda_1}$ is proposed under stratified population as

$$\hat{\lambda_1} = \sum_{h=1}^{L} W_h \hat{\lambda_{h1}}$$

where, $W_h = \dfrac{nh}{N}$
and

$$E(\hat{\lambda_1}) = E\left[\sum_{h=1}^{L} W_h \hat{\lambda_{h1}}\right] = \lambda_1$$

which can be proved if we suppose that $y_{h1}, y_{h2}, ...y_{hnh}$ are i.i.d. Poisson variate with parameter λ_{h0}, then, we have

$$E(\hat{\lambda_1}) = E\left[\sum_{h=1}^{L} W_h \hat{\lambda_{h1}}\right]$$

$$\begin{aligned} &= \sum_{h=1}^{L} \frac{W_h}{T_h+(1-T_h)P_h+(1-T_h)\dfrac{(1-\pi_{h1})}{\pi_{h1}}(1-P_h)Q_{1h}} \\ &+ \left[\frac{1}{nh}\sum_{i=1}^{nh} E(y_{hi}) - (1-T_h)\frac{(1-\pi_{h1})}{\pi_{h1}}(1-P_h)Q_{2h}\lambda_{h2}]\right] \\ &= \sum_{h=1}^{L} \frac{W_h}{T_h+(1-T_h)P_h+(1-T_h)\dfrac{(1-\pi_{h1})}{\pi_{h1}}(1-P_h)Q_{1h}} \\ &+ \left[\frac{1}{nh}\sum_{i=1}^{nh} \lambda_{h0} - (1-T_h)\frac{(1-\pi_{h1})}{\pi_{h1}}(1-P_h)Q_{2h}\lambda_{h2}]\right]\end{aligned}$$

$$E(\hat{\lambda_1}) = \left[\sum_{h=1}^{L} W_h \lambda_{h1}\right] = \lambda_1 \tag{8}$$

with the variance,

$$V(\hat{\lambda_1}) = \sum_{h=1}^{L} W_h \frac{\lambda_{h1}}{n_h(T_h + (1-T_h)P_h + (1-T_h)\frac{(1-\pi_{h1})}{\pi_{h1}}(1-P_h)Q_{1h})} + \frac{(1-T_h)\frac{(1-\pi_{h1})}{\pi_{h1}}(1-P_h)Q_{2h}\lambda_{h2})}{n_h(T_h + (1-T_h)P_h + (1-T_h)\frac{(1-\pi_{h1})}{\pi_{h1}}(1-P_h)Q_{1h})^2} \tag{9}$$

We can brief it if we put $y_{h1}, y_{h2}, ...y_{hnh}$ are i.i.d. Poisson variate with parameter λ_{h0} and samples are drawn independently from different strata, therefore , we have

$$V(\hat{\lambda_1}) = V\left[\sum_{h=1}^{L} W_h \hat{\lambda_{h1}}\right]$$

$$V(\hat{\lambda_1}) = \left[\sum_{h=1}^{L} W_h^2 V(\hat{\lambda_{h1}})\right]$$

$$V(\hat{\lambda_1}) = \sum_{h=1}^{L} \frac{W_h^2}{(T_h + (1-T_h)P_h + (1-T_h)\frac{(1-\pi_{h1})}{\pi_{h1}}(1-P_h)Q_{1h})^2} \left[\frac{1}{n_h^2}\sum_{i=1}^{nh}\lambda_{h0}\right]$$

Hence, after simplification we have

$$V(\hat{\lambda_1}) = \sum_{h=1}^{L} W_h \frac{\lambda_{h1}}{n_h(T_h + (1-T_h)P_h + (1-T_h)\frac{(1-\pi_{h1})}{\pi_{h1}}(1-P_h)Q_{1h})} + \frac{(1-T_h)\frac{(1-\pi_{h1})}{\pi_{h1}}(1-P_h)Q_{2h}\lambda_{h2})}{n_h(T_h + (1-T_h)P_h + (1-T_h)\frac{(1-\pi_{h1})}{\pi_{h1}}(1-P_h)Q_{1h})^2}$$

and the unbiased estimate of the variance of the proposed estimator will be

$$V(\hat{\lambda_1}) = \sum_{h=1}^{L} \frac{1}{n_h^2(T_h + (1-T_h)P_h + (1-T_h)\frac{(1-\pi_{h1})}{\pi_{h1}}(1-P_h)Q_{1h})^2}$$

$$\left[\sum_{i=1}^{nh} \lambda_{hi}]\right] \tag{10}$$

By taking expectation both sides of equation (10). We may easily prove that $\hat{V}(\hat{\lambda_1})$ is an unbiased estimate of $V(\hat{\lambda_1})$ by utilizing $E(y_{ij}) = \lambda_{h0}$ as y_{ij} follows $P(\lambda_{h0})$. The precision of the proposed estimator under stratified random sampling depends upon the selection of sample size n_h from h^{th} stratum ($h = 1, 2, ..., L$).
The allocation method for selection of sample from different strata is based on the availability of prior information of stratum variance.

2.1. Proportional allocation. Under the proportional allocation the sample size $n_h = nW_h$, where, $n = \sum_{h=1}^{L} n_h$ is the size of total sample drawn from L strata. The variance of the proposed estimator under proportional allocation is obtained as

$$\begin{aligned} V(\hat{\lambda_1})_P &= \frac{1}{n}\sum_{h=1}^{L} W_h \frac{\lambda_{h1}}{(T_h + (1-T_h)P_h + (1-T_h)\frac{(1-\pi_{h1})}{\pi_{h1}}(1-P_h)Q_{1h})} \\ &+ \frac{(1-T_h)\frac{(1-\pi_{h1})}{\pi_{h1}}(1-P_h)Q_{2h}\lambda_{h2}}{(T_h + (1-T_h)P_h + (1-T_h)\frac{(1-\pi_{h1})}{\pi_{h1}}(1-P_h)Q_{1h})^2} \end{aligned} \tag{11}$$

2.2. Optimum allocation. In this method, the size of the sample to be drawn from the h^{th} stratum is derived using the following cost function

$$C = C_0 + \sum_{h=1}^{L} n_h C_h$$

Where C_0 denotes the overhead cost and C_h be the survey cost per unit in the h^{th} stratum. Sample size n_h from the h^{th} stratum is as follows

$$n_h = n\frac{W_h\sqrt{\zeta_h}C_h}{\sum_{h=1}^{L} W_h\sqrt{\zeta_h}C_h}$$

Under optimum allocation variance of the proposed estimator is given as

$$V(\hat{\lambda_1})_{opt} = \frac{1}{n}\left[\sum_{h=1}^{L} W_h\sqrt{\zeta_h C_h}\right]\left[\sum_{h=1}^{L} W_h\sqrt{\zeta_h C_h}\right] \tag{12}$$

$$\hat{\zeta}_h = \frac{\lambda_{h1}}{(T_h + (1-T_h)P_h + (1-T_h)\frac{(1-\pi_{h1})}{\pi_{h1}}(1-P_h)Q_{1h})} + \frac{(1-T_h)\frac{(1-\pi_{h1})}{\pi_{h1}}(1-P_h)Q_{2h}\lambda_{h2}}{(T_h + (1-T_h)P_h + (1-T_h)\frac{(1-\pi_{h1})}{\pi_{h1}}(1-P_h)Q_{1h})^2} \tag{13}$$

TABLE 1. Percent relative efficiency of the proposed estimator with respective to Lee et al. [16], when the proportion of the rare unrelated non-sensitive is known ($W_1 + W_2 = 1$).

W_1	λ_1	Q_1	Q_2	Q_3	$\lambda_2 = 0.9$	$\lambda_2 = 0.7$	$\lambda_2 = 0.5$	$\lambda_2 = 0.3$	$\lambda_2 = 0.1$
0.5	2.5	0.3	0.3	0.4	113.2093	113.6123	114.0403	114.4958	114.9815
0.5	2.6	0.3	0.3	0.4	113.2773	113.6684	114.0829	114.523	114.9912
0.5	2.7	0.3	0.3	0.4	113.3409	113.7208	114.1225	114.5483	115.0002
0.5	2.8	0.3	0.3	0.4	113.4006	113.7697	114.1595	114.5718	115.0085
0.5	2.9	0.3	0.3	0.4	113.4566	113.8157	114.1942	114.5938	115.0162
0.5	2.5	0.2	0.4	0.4	110.0295	110.4842	110.9635	111.4693	112.0039
0.5	2.6	0.2	0.4	0.4	110.1066	110.5473	111.0109	111.4993	112.0145
0.6	2.5	0.3	0.3	0.4	115.6223	116.1722	116.7604	117.3911	118.069
0.6	2.6	0.3	0.3	0.4	115.7149	116.249	116.8191	117.4288	118.0826
0.6	2.7	0.3	0.3	0.4	115.8016	116.3208	116.8738	117.464	118.0951
0.6	2.8	0.3	0.3	0.4	115.8829	116.3881	116.925	117.4967	118.1068
0.6	2.9	0.3	0.3	0.4	115.9593	116.4511	116.9728	117.5273	118.1177
0.6	2.5	0.2	0.4	0.4	111.7797	112.3497	112.9533	113.5937	114.2742
0.6	2.6	0.2	0.4	0.4	111.8761	112.4289	113.0133	113.6318	114.2877
0.7	2.5	0.3	0.3	0.4	116.9891	117.6299	118.3183	119.0596	119.8601
0.7	2.6	0.3	0.3	0.4	117.0968	117.7197	118.3872	119.1041	119.8762
0.7	2.7	0.3	0.3	0.4	117.1977	117.8036	118.4514	119.1455	119.891
0.7	2.8	0.3	0.3	0.4	117.2925	117.8822	118.5114	119.1841	119.9048
0.7	2.9	0.3	0.3	0.4	117.3815	117.956	118.5676	119.2201	119.9177
0.7	2.5	0.2	0.4	0.4	112.7445	113.3818	114.0585	114.7785	115.5459
0.7	2.6	0.2	0.4	0.4	112.8521	113.4705	114.1258	114.8214	115.5612
0.8	2.5	0.3	0.3	0.4	117.6875	118.3771	119.1193	119.9204	120.7876
0.8	2.6	0.3	0.3	0.4	117.8034	118.4738	119.1937	119.9685	120.805
0.8	2.7	0.3	0.3	0.4	117.9119	118.5642	119.263	120.0133	120.8211

0.8	2.8	0.3	0.3	0.4	118.0138	118.649	119.3278	120.0551	120.8361
0.8	2.9	0.3	0.3	0.4	118.1097	118.7285	119.3885	120.0941	120.8501
0.8	2.5	0.2	0.4	0.4	113.2304	113.9026	114.6174	115.3789	116.1919
0.8	2.6	0.2	0.4	0.4	113.3439	113.9963	114.6885	115.4244	116.2081
0.9	2.5	0.3	0.3	0.4	117.9897	118.7008	119.4669	120.2946	121.1915
0.9	2.6	0.3	0.3	0.4	118.1091	118.8006	119.5437	120.3444	121.2095
0.9	2.7	0.3	0.3	0.4	118.221	118.8939	119.6153	120.3907	121.2262
0.9	2.8	0.3	0.3	0.4	118.3261	118.9814	119.6823	120.4338	121.2417
0.9	2.9	0.3	0.3	0.4	118.425	119.0634	119.745	120.4742	121.2562
0.9	2.5	0.2	0.4	0.4	113.4391	114.1266	114.8579	115.6376	116.4706
0.9	2.6	0.2	0.4	0.4	113.5551	114.2224	114.9307	115.6842	116.4872

3. Conclusion

This manuscript highlighted the significance of randomized response technique in survey methodology. By leveraging the known proportion of a non-sensitive attribute, researchers can enhance the precision and reliability of their estimates for the sensitive attribute. This innovative approach holds promise for situations where individuals are reluctant to disclose certain information openly. Our exploration underscores the importance of nuanced survey techniques in capturing hidden dimensions of human behavior and characteristics. It also exemplifies the potential of leveraging existing data to refine estimation processes. As we continue to grapple with the challenges of gathering accurate and comprehensive data, this research contributes valuable insights to the field, offering a strategic framework for future studies seeking to estimate sensitive attributes in a population while considering the known proportions of unrelated non-sensitive attributes. Further, this research opens new avenues for survey methodology and provides a methodological bridge between sensitive and non-sensitive attributes in the estimation process, offering practical applications and avenues for further exploration in the ever-evolving landscape of social research and data collection.

Conflicts of interest : The authors declare no conflict of interest.

Data availability : Not applicable

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