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Characterization and Estimation of Generalized Inverse Power Lindley Distribution

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***Abstract*—One of the most important branch of statistics is survival and reliability analysis. There are various lifetime models available in literature that have applications in these fields. However the researchers always keep searching for more flexible models that are effective in more complex situations. With the same motivation, an effort has been made to introduce a new distribution named as Generalized Inverse Power Lindley distribution that is expected to turnout more constructive while dealing with complex real life data. Various statistical properties of the model have been derived. The parameter estimates are obatined using Maximum Likelihood Estimation (MLE) technique. Simulation study has been conducted to assess the performance of maximum likleihood estimators. Applicability of the proposed model to the real data has been investigated by comparing the model with some existing distributions.**

***Index Terms*—Exponentiation, Simulation, Quantile function, Order statistics, Stochastic ordering**

I. Introduction

Lindley distribution (LD) introduced by Lindley (1958) has drawn a lot of attention from researchers because of its broad applications in modeling the data having monotone hazard rates. A Random Variable (RV) Z is said to have LD if its probability density function (pdf) is given by

$$f(z,\beta)=\frac{\beta^2}{1+\beta}\left(1+z\right)e^{-\beta z}\ ;\ z>0,\beta>0 \tag{1}$$

LD has been extended by various researchers including Ghitany et al. (2008), Nadarajah et al. (2011), Ghitany et al. (2013).

It has been observed that most of the real-life systems have non-monotone(bathtub (BT) and upside down bathtub (UBT)) hazard rates. For the analysis of lifetime data having BT shape hazard functions, several lifetime models have been introduced by many authors (see Mudholkar et al. (1993), Xie et al. (1996), Xie et al. (2002)). Interestingly, the inverse class of the probability models turn out very useful for modeling UBT shape hazard functions. Sharma et al. (2015) introduced Inverse Lindley distribution (ILD) with pdf given by:

$$f(z,\beta)=\frac{\beta^2}{1+\beta}\left(\frac{1+z}{z^3}\right)e^{\frac{-\beta}{z}}\ ;\ z>0,\beta>0 \tag{2}$$

Sharma et al. (2016) extended ILD by adding a parameter and obtained a Generalized Inverse Lindley Distribution (GILD). Note that Barco et al. (2016) also generalized ILD by taking the transformation $Z=Y^{\frac{1}{\alpha}}$ where Y follows ILD. The pdf of GILD is given by:

$$f(z,\beta)=\frac{\alpha\beta^2}{1+\beta}\left(\frac{1+z^\alpha}{z^{2\alpha+1}}\right)e^{\frac{-\beta}{z^\alpha}}\ ;\ z>0,\alpha>0,\beta>0 \tag{3}$$

In this article a new three parameter distribution named as Generalized Inverse Power Lindley Distribution (GIPLD) has been introduced. The proposed distribution is obtained by using the transformation $H(z)=[G(z)]^\theta$ where $G(z)$ is a CDF and θ is a positive real number. The new distribution thus obtained involves GILD and ILD as its sub-models for $\theta=1$ and $\alpha=\theta=1$ respectively. A RV Z is said to follow GIPLD if its cumulative distribution function (CDF) is given by:

$$G(z)=\left[\left(1+\frac{\beta}{1+\beta}\frac{1}{z^\alpha}\right)e^{\frac{-\beta}{z^\alpha}}\right]^\theta;\quad z>0\ s(\alpha,\,\beta,\,\theta)>0 \tag{4}$$

and the corresponding pdf as:

$$g(z)=\frac{\alpha\beta^2\theta}{1+\beta}\left(\frac{1+z^\alpha}{z^{2\alpha+1}}\right)e^{\frac{-\theta\beta}{z^\alpha}}\left[1+\frac{\beta}{1+\beta}\frac{1}{z^\alpha}\right]^{\theta-1};\quad z>0\,,\,(\alpha,\,\beta,\,\theta)>0 \tag{5}$$

where (α,β,θ) are the parameters of the distribution, β being scale while as α,θ are the shape parameters.

The aim of this article is to obtain a more flexible model that exhibits both monotone and non-monotone behavior and thus motivates us to apply single model for two distinct behaviors

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