Poisson Ishita Distribution: A New Compounding Probability Model

¹Anwar Hassan, ²Showkat Ahmad Dar, ³Peer Bilal Ahmad

^{1,2}Department of Statistics, University of Kashmir, Srinagar (J&K), India. ³Department of Mathematical Sciences, Islamic University of Science & Technology, Awantipora, Pulwama (J&K), India Corresponding Author: Anwar Hassan

Abstract: In this article, we propose a new probability distribution by compounding Poisson distribution with Ishita distribution. Important mathematical and statistical properties of the distribution have been derived and discussed. The expressions for coefficient of variation, skewness, kurtosis, reliability analysis and order statistics has been obtained. Then, parameter estimation is discussed using maximum likelihood method of estimation. Finally, real data set is analyzed to investigate the suitability of the proposed distribution in modeling count dataset representing epileptic seizure counts.

Keywords: Poisson distribution, Ishita distribution, compound distribution, Count data, Maximum likelihood estimation.

Date of Submission: 15-02-2019	Date of acceptance: 04-03-2019

I. INTRODUCTION

Compounding of probability distributions is a sound and innovative technique to obtain new probability distributions to fit data sets not adequately fit by common parametric distributions. Compound distributions serve well to describe various phenomena in biology, epidemiology, genetics and so on. Mathematically, compound distribution arises when all or some parameters of a distribution known as parent distribution vary according to some probability distribution called the compounding distribution, for instance negative binomial distribution can be obtained from Poisson distribution when its rate parameter follows gamma distribution. If the parent distribution is discrete/continuous then resultant compound distribution will also be discrete/continuous respectively i.e., the support of the parent distribution determines the support of compound distributions. The work has been done in this particular area since 1920. It is well known that Greenwood and Yule(1920) established a relationship between Poisson distribution and a negative binomial distribution through compounding mechanism by treating the rate parameter in Poisson distribution as gamma variate. Skellam(1948) derived a probability distribution from the binomial distribution by regarding the probability of success as a beta variable between sets of trials. Lindely (1958) suggested a one parameter distribution to illustrate the difference between fiducial distribution and posterior distribution. Dubey (1970) derived a compound gamma, beta and F distribution by compounding a gamma distribution with another gamma distribution and reduced it to the beta Ist and beta 2^{nd} kind and to the F distribution by suitable transformations. Gerstenkorn(1993,1996) proposed several compound distributions, he obtained compound of gamma distribution with exponential distribution by treating the parameter of gamma distribution as an exponential variate and also obtained compound of polya with beta distribution. Mahmoudi et al. (2010) generalized the Poisson-Lindely distribution of Sankaran (1970) and showed that their generalized distribution has more flexibility in analyzing count data. Zamani and Ismail (2010) constructed a new compound distribution by compounding negative binomial with one parameter Lindley distribution that provides good fit for count data where the probability at zero has a large value. A new generalized negative binomial distribution was proposed by Gupta and Ong (2004), this distribution arises from Poisson distribution if the rate parameter follows generalized gamma distribution; the resulting distribution so obtained was applied to various data sets and can be used as better alternative to negative binomial distribution. Rashid, Ahmad and Jan (2016) proposed a new competitive count data model, by compounding negative binomial distribution with Kumaraswamy distribution that finds its application in biological sciences. Para and Jan (2018) introduced two compounding models with applications to handle count data in medical sciences.

In this paper we propose a new compounding distribution by compounding Poisson distribution with Ishita distribution, as there is a need to find more flexible model for analyzing statistical data.