Negative Binomial Weighted Exponential Distributionn and Its Applications

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ABSTRACT: In this paper, we obtained a new model for count data by compounding of Negative Binomial distribution(NBD) with Weighted Exponential Distribution (WED). Important mathematical and statistical properties of the distribsution have been derived and discussed. Then, parameter estimation is discussed using maximum likelihood method of estimation. Finally, real data set is analyzed to investigate the suitability of the proposed distribution in modeling count data.

KEYWORDS: Negative Binomial Distribution, Weighted Exponential distribution, compound distribution, count data.

1.Introduction

The modeling of count data is one of the most important topic in various fields of real life like actuarial science, insurance, transport and health and so on. Researchers have proved that mixed Poisson and mixed Negative distributions provides better fit to count data as compared to old existing models. Altun (2019) introduced Poisson cousil lindly regression model for analyzing over dispersed count data. Chesneau et al (2020) introduced Cosine geometric distribution for count data modeling. Malete (2017) introduced a new count data model by compounding Negative binomial and generalized Exponential distribution. Deepesh Bhati (2016) introduced a new life time model by mixing Lindley and Exponential distribution. Mohamed E Ghitany (2008) introduced Size biased Poisson Lindley distribution and its applications and finds its applications in immunogold assay and abundance data. K.M. Sakthivel (2017) obtained Zero Inflated Negative Binomial Sushila for count data model and fits its applications in automobile insurance policies. Christopher S. withers (2011) introduced a new count data model by compounding Poisson and Gamma distribution. Tadeusz Gerstenkorn (2004) obtained a compound of generalized negative binomial distribution with the generalized beta distribution. Nahmias and demmy (1982) obtained a logarithmic version and finds its application to model lead time demand. Gerstenkorn (1993, 1996) derived a new count data model by compounding generalized gamma distribution with exponential distribution. And also proposed a compound of polya with beta distribution .Yupapian Atikankul (2020) introduces a new count data model by mixing Poisson distribution with weighted Lindley distribution and finds its application to insurance claims data.

In this paper we propose a new count data model by compounding Negative Binomial With weighted Exponential distribution, as there is a need to find more flexible model for analyzing statistical data

2. Construction of the Negative Binomial-Weighted Exponential distribution

Probability mass function (pmf) of Negative Binomial distribution (NBD) is given by

$$g(z) = \binom{r+z-1}{z} p^r q^z \,; z = 0,1,2\ldots, r > 0, and\; 0 < p < 1$$

The factorial moments of NBD is given by

$$\mu_{[k]} = \frac{\Gamma r + k}{\Gamma r}\left(\frac{q}{p}\right)^k, \;\; k = 1,2\ldots$$

$$E(Z) = \frac{rq}{p} \;\; and \;\; V(Z) = \frac{rq}{p^2}$$

The Weighted exponential distribution is a newly proposed lifetime model formulated by Gomez et al. (2014) i.e, WED (α, β) with the Probability density function (pdf) is given by

$$h(Z;\alpha,\theta) = \frac{\alpha^2(1+\beta z)e^{-\alpha z}}{\alpha+\beta}, \; \alpha, \beta > 0, z > 0$$

Moment generating function (MGF) of WED is given by

$$M_z(t) = \frac{\alpha^2}{\alpha+\beta}\left(\frac{1}{(\alpha-t)} + \frac{\beta}{(\alpha-t)^2}\right)$$

3 Definition of Proposed Model (Negative Binomial Weighted Exponential Distribution)

If $Z|\lambda \sim$ NB$\left(r, p = e^{-\lambda}\right)$, λ is itself a random variable following weighted exponential distribution with parameter α, β, then the resulting distribution by marginalizing over λ will be known as negative binomial weighted exponential distribution, which is denoted by $NBWED(Z; r, \alpha\beta)$.The proposed distribution is discrete as the parent distribution NBD is discrete.

Theorem3.1: Let Z ~ NBWED $(Z; r, \alpha \; and \; \beta)$ .Then the pmf of s given by

$$\text{P(Z=z)=}\binom{z+r-1}{z}\sum_{j=0}^{z}\binom{z}{j}(-1)^j \frac{\alpha^2}{(\alpha+\beta)}\left(\frac{(r+j+\alpha)+\beta}{(r+j+\alpha)^2}\right)$$

Proof: If $Z/\lambda \sim NBD(r, p = e^{-\lambda}) \, and \, \lambda \sim WED(\alpha, \beta)$, the pmf of Z can be obtained as

$$g(z/\lambda) = \binom{r+z-1}{z} e^{-\lambda r}(1-e^{-\lambda})^z$$

And

$$h(\lambda;\alpha,\beta)=\frac{\alpha^2(1+\beta x)e^{-\alpha z}}{\alpha+\beta}$$

We know that

$$P(Z=z)=\int_0^\infty g(z/\lambda)h(\lambda;\theta,\alpha)d\lambda$$

$$P(Z=z)=\int_0^\infty \binom{z+r-1}{z}e^{-\lambda r}(1-e^{-\lambda})^z\,\frac{\alpha^2(1+\beta x)e^{-\alpha z}}{\alpha+\beta}d\lambda$$

$$P(Z=z)=\frac{\alpha^2}{\alpha+\beta}\binom{z+r-1}{z}\sum_{j=0}^{z}\binom{z}{j}(-1)^j\int_0^\infty e^{-\lambda(r+j+\alpha)}(1+\beta\lambda)d\lambda$$

$$P(Z=z)=\binom{z+r-1}{z}\sum_{j=0}^{z}\binom{z}{j}(-1)^j\,\frac{\alpha^2}{(\alpha+\beta)}\left(\frac{(r+j+\alpha)+\beta}{(r+j+\alpha)^2}\right),\qquad z>0,\beta,\alpha,r>0$$

Which is the pmf of NBWED

Corollary3.1: If we put $\beta=0$ the NBWED reduces to Negative Binomial Exponential distribution with pmf as

$$P_1(Z=z)=\binom{z+r-1}{z}\sum_{j=0}^{z}\binom{z}{j}(-1)^j\left(\frac{1}{(r+j+\alpha)}\right),\qquad z>0,\alpha,r>0$$

Proof: If $Z/\lambda \sim NBD(r,p=e^{-\lambda})\, and\, \lambda \sim ED(\alpha)$, the pmf of Z can be obtained as

$$P_1(Z=z)=\int_0^\infty g(z/\lambda)h(\lambda;\alpha)d\lambda$$

$$P_1(Z=z)=\binom{z+r-1}{z}\sum_{j=0}^{z}\binom{z}{j}(-1)^j\left(\frac{1}{(r+j+\alpha)}\right),\qquad z>0,\alpha,r>0$$

Corollary3.2: If we put r=1 and $\beta=0$ the NBWED reduces to Geometric Exponential Distribution with pmf as below

$$P_2(Z=z)=\left(\frac{\alpha\sum_{j=0}^{z}\binom{z}{j}(-1)^j}{(1+j+\theta)}\right),\qquad z>0,\alpha>0$$

Proof: If $Z/\lambda \sim NBD(p=e^{-\lambda})\, and\, \lambda \sim ED(\alpha)$, the pmf of Z can be obtained as

$$P_2(Z=z)=\int_0^\infty g(z/\lambda)\,h(\lambda;\alpha)\,d\lambda$$

$$P_2(Z=z)=\left(\frac{\alpha\sum_{j=0}^{z}\binom{z}{j}(-1)^j}{(1+j+\theta)}\right), \qquad z>0, \alpha>0$$

4. Mean and Variance:

The men and variance of NBWED can be obtained by using the property of conditional mean and variance

(i)Conditional expectation identity $E(Z^k)=E_\lambda(z^k\,|\,\lambda)$

Since $Z\,|\,\lambda \sim NBD(r, p=e^{-\lambda})$ where λ is itself a random variable following $QAD \sim (\alpha,\beta)$, therefore we have

$$E(Z^k)=E_\lambda\left(Z^k\,|\,\lambda\right)$$

$$E(Z^k)=r\,E_\lambda\left(\frac{1-e^{-\lambda}}{e^{-\lambda}}\right)^k$$

$$E(Z^k)=r\sum_{j=0}^{k}\binom{k}{j}(-1)^j E_\lambda(e^{\lambda(k-j)})$$

$$E(Z^k)=r\sum_{j=0}^{k}\binom{k}{j}(-1)^j\int_0^\infty e^{\lambda(k-j)}\,\frac{\alpha^2(1+\beta\lambda)e^{-\alpha\lambda}}{\alpha+\beta}\,d\lambda$$

$$E(Z^k)=r\sum_{j=0}^{k}\binom{k}{j}(-1)^j\,\frac{\alpha^2}{(\alpha+\beta)}\int_0^\infty e^{-\lambda(-k+j+\alpha)}(1+\beta\lambda)\,d\lambda$$

Put k=1 we get mean of NBQAD

$$E(Z)=r\,\frac{\alpha^2}{\alpha+\beta}\left(\frac{(\alpha+j-k)+\beta}{(\alpha+j-k)^2}\right)$$

(ii) Conditional expectation identity V(Z)= $E_\lambda(V(Z\,|\,\lambda))+V_\lambda(E(Z\,|\,\lambda))$

$$V(Z)=E\left(\frac{rq}{p^2}\right)+V\left(\frac{rq}{p}\right)$$

$$V(Z)=r(r-1)\frac{\alpha^2}{\alpha+\beta}\left(\frac{(\alpha-2)+\beta}{(\alpha-2)^2}-2\frac{(\alpha-1)+\beta}{(\alpha-1)^2}+\frac{\alpha+\beta}{\alpha^2}\right)+r\left(\frac{\alpha^2}{\alpha+\beta}\left|\frac{(\alpha-1)+\beta}{(\alpha-1)^2}-\right|\frac{\alpha+\beta}{\alpha^2}\right)-\left(r\frac{\alpha^2}{\alpha+\beta}\left(\frac{(\alpha+j-k)+\beta}{(\alpha+j-k)^2}\right)\right)^2$$

5. Moments of NBQAD

5.1 factorial moments:

The factorial moments of order K of NBQAD is

$$m_k(Z\mid\lambda)=\frac{\Gamma r+k}{\Gamma r}\left(\frac{1-e^{-\lambda}}{e^{-\lambda}}\right)^k$$

$\lambda\, is\, it\, self\;\, a\, random\, \mathrm{var}\, iale\;\, following\, QAD$

$$\mu_{[k]}(Z)=E_\lambda(m_k(Z\mid\lambda))$$

$$\mu_{[k]}(Z)=\frac{\Gamma r+k}{\Gamma r}E_\lambda\left(\frac{1-e^{-\lambda}}{e^{-\lambda}}\right)^k$$

$$\mu_{[k]}(Z)=\frac{\Gamma r+k}{\Gamma r}\sum_{j=0}^{k}\binom{k}{j}(-1)^j E(e^{\lambda(k-j)})$$

$$\mu_{[k]}(Z)=\frac{\Gamma r+k}{\Gamma r}\sum_{j=0}^{k}\binom{k}{j}(-1)^j\frac{\alpha^2}{\alpha+\beta}\left(\frac{\alpha-(k-j)+\beta}{(\alpha-(k-j))^2}\right)$$

Put k=1,2,3 and 4 we get the first four factorial moments are obtained as under

$$\mu_{[1]}(Z)=r\delta_0(\delta_1-\delta_2)$$

$$\mu_{[2]}(Z)=r(r+1)\delta_0(\delta_3-2\delta_1+\delta_2)$$

$$\mu_{[3]}(Z)=r(r+1)(r+2)\delta_0(\delta_4-3\delta_3+3\delta_1-\delta_2)$$

$$\mu_{[4]}(Z)=r(r+1)(r+2)(r+3)\delta_0(\delta_5-4\delta_4+6\delta_3-4\delta_1+\delta_2)$$

$$Where\;\delta_0=\frac{\alpha^2}{\alpha+\beta},\delta_1=\frac{(\alpha-1)+\beta}{(\alpha-1)^2},\delta_2=\frac{\alpha+\beta}{\alpha^2},\delta_3=\frac{(\alpha-2)+\beta}{(\alpha-2)^2},$$

$$\delta_4 = \frac{(\alpha-3)+\beta}{(\alpha-3)^3} \, and \, \delta_5 = \frac{(\alpha-4)+\beta}{(\alpha-4)^2}$$

5.2 Moments about origin (Raw moments):

The first four moments about origin are obtained as under

$$\mu_1' = r\delta_0(\delta_1 - \delta_2)$$

$$\mu_2' = r(r+1)\delta_0(\delta_3 - 2\delta_1 + \delta_2) + r\delta_0(\delta_1 - \delta_2)$$

$$\mu_3' = r(r+1)(r+2)\delta_0(\delta_4 - 3\delta_3 + 3\delta_1 - \delta_2) + 3(r(r+1)\delta_0(\delta_3 - 2\delta_1 + \delta_2)) + r\delta_0(\delta_1 - \delta_2)$$

$$\mu_4' = r(r+1)(r+2)(r+3)\delta_0(\delta_5 - 4\delta_4 + 6\delta_3 - 4\delta_1 + \delta_2) + 6(r(r+1)(r+2)\delta_0(\delta_4 - 3\delta_3 + 3\delta_1 - \delta_2)) + 7(r(r+1)\delta_0(\delta_3 - 2\delta_1 + \delta_2)) + r\delta_0(\delta_1 - \delta_2)$$

$$Where \, \delta_0 = \frac{\alpha^2}{\alpha+\beta}, \delta_1 = \frac{(\alpha-1)+\beta}{(\alpha-1)^2}, \delta_2 = \frac{\alpha+\beta}{\alpha^2}, \delta_3 = \frac{(\alpha-2)+\beta}{(\alpha-2)^2},$$

$$\delta_4 = \frac{(\alpha-3)+\beta}{(\alpha-3)^3} \, and \, \delta_5 = \frac{(\alpha-4)+\beta}{(\alpha-4)^2}$$

5.3 Moments about mean (central moments):

The moments about mean are obtained as under

$$\mu_2 = r(r+1)\delta_0(\delta_3 - 2\delta_1 + \delta_2) + r\delta_0(\delta_1 - \delta_2) - (r\delta_0(\delta_1 - \delta_2))^2$$

$$\mu_3 = r(r+1)(r+2)\delta_0(\delta_4 - 3\delta_3 + 3\delta_1 - \delta_2) + 3(r(r+1)\delta_0(\delta_3 - 2\delta_1 + \delta_2)) + r\delta_0(\delta_1 - \delta_2) - 3 (r(r+1)\delta_0(\delta_3 - 2\delta_1 + \delta_2) + r\delta_0(\delta_1 - \delta_2))(r\delta_0(\delta_1 - \delta_2)) + 2(r\delta_0(\delta_1 - \delta_2))^3$$

$$\mu_4 = r(r+1)(r+2)(r+3)\delta_0(\delta_5 - 4\delta_4 + 6\delta_3 - 4\delta_1 + \delta_2) - 4(r(r+1)(r+2)\delta_0(\delta_4 - 3\delta_3 + 3\delta_1 - \delta_2) + 3(r(r+1)\delta_0(\delta_3 - 2\delta_1 + \delta_2)) + r\delta_0(\delta_1 - \delta_2))(r\delta_0(\delta_1 - \delta_2)) + 6(r(r+1)\delta_0(\delta_3 - 2\delta_1 + \delta_2) + r\delta_0(\delta_1 - \delta_2))(r\delta_0(\delta_1 - \delta_2))^2 - 3(r\delta_0(\delta_1 - \delta_2)^4$$

$$Where \, \delta_0 = \frac{\alpha^2}{\alpha+\beta}, \delta_1 = \frac{(\alpha-1)+\beta}{(\alpha-1)^2}, \delta_2 = \frac{\alpha+\beta}{\alpha^2}, \delta_3 = \frac{(\alpha-2)+\beta}{(\alpha-2)^2},$$

$$\delta_4 = \frac{(\alpha-3)+\beta}{(\alpha-3)^3} \, and \, \delta_5 = \frac{(\alpha-4)+\beta}{(\alpha-4)^2}$$

6. Coefficient of variation, Skewness, kurtosis and Index of Disperation:

$$Coefficient\, of\, \mathrm{var}ition(C.V) = \frac{\sigma}{\mu_1'} = \frac{\sqrt{r(r+1)\delta_0(\delta_3 - 2\delta_1 + \delta_2) + r\delta_0(\delta_1 - \delta_2) - (r\delta_0(\delta_1 - \delta_2))^2}}{r\delta_0(\delta_1 - \delta_2)}$$

$$Coefficient\ of\ Skewness(\sqrt{\beta_1}) = \frac{\mu_3}{\mu_2^{3|2}} = \frac{r(r+1)(r+2)\delta_0(\delta_4 - 3\delta_3 + 3\delta_1 - \delta_2) + 3(r(r+1)\delta_0(\delta_3 - 2\delta_1 + \delta_2)) + r\delta_0(\delta_1 - \delta_2) - 3 (r(r+1)\delta_0(\delta_3 - 2\delta_1 + \delta_2) + r\delta_0(\delta_1 - \delta_2))(r\delta_0(\delta_1 - \delta_2)) + 2(r\delta_0(\delta_1 - \delta_2))^3}{(r(r+1)\delta_0(\delta_3 - 2\delta_1 + \delta_2) + r\delta_0(\delta_1 - \delta_2) - (r\delta_0(\delta_1 - \delta_2))^2)^{3|2}}$$

$$Coefficient\ of\ kurtosis(\beta_2) = \frac{\mu_4}{\mu_2^{\,2}} = \frac{\begin{array}{l} r(r+1)(r+2)(r+3)\delta_0(\delta_5 - 4\delta_4 + 6\delta_3 - 4\delta_1 + \delta_2) - 4(r(r+1)(r+2) \\ \delta_0(\delta_4 - 3\delta_3 + 3\delta_1 - \delta_2) + 3(r(r+1)\delta_0(\delta_3 - 2\delta_1 + \delta_2)) \\ + r\delta_0(\delta_1 - \delta_2))(r\delta_0(\delta_1 - \delta_2)) + 6(r(r+1)\delta_0(\delta_3 - 2\delta_1 + \delta_2) \\ + r\delta_0(\delta_1 - \delta_2))(r\delta_0(\delta_1 - \delta_2))^2 - 3(r\delta_0(\delta_1 - \delta_2)^4 \end{array}}{(r(r+1)\delta_0(\delta_3 - 2\delta_1 + \delta_2) + r\delta_0(\delta_1 - \delta_2) - (r\delta_0(\delta_1 - \delta_2))^2)^2}$$

$$Index of\ dispertion(\gamma) = \frac{\sigma^2}{\mu_1'} = \frac{r(r+1)\delta_0(\delta_3 - 2\delta_1 + \delta_2) + r\delta_0(\delta_1 - \delta_2) - (r\delta_0(\delta_1 - \delta_2))^2}{r\delta_0(\delta_1 - \delta_2)}$$

$$Where\ \delta_0 = \frac{\alpha^2}{\alpha+\beta}, \delta_1 = \frac{(\alpha-1)+\beta}{(\alpha-1)^2}, \delta_2 = \frac{\alpha+\beta}{\alpha^2}, \delta_3 = \frac{(\alpha-2)+\beta}{(\alpha-2)^2},$$

$$\delta_4 = \frac{(\alpha-3)+\beta}{(\alpha-3)^3}\ and\ \delta_5 = \frac{(\alpha-4)+\beta}{(\alpha-4)^2}$$

7. MAXIMUM LIKELIHOOD ESTIMATION

Method of Maximum Likelihood Estimation is simple and most efficient method of estimation. Let $Z_1, Z_2 ... Z_n$ be the random size of sample n drawn from NBWED, then the log likelihood function of NBWED is given as

$$LogL = \sum_{i=1}^{n} \binom{z+r-1}{z} + 2n\log\alpha - n\log(\alpha+\beta) + \sum_{i=1}^{n}\left(\sum_{j=0}^{z}\binom{z}{j}(-1)^j\left(\frac{(r+j+\alpha)+\beta}{(r+j+\alpha)^2}\right)\right)$$

$$\frac{\delta}{\delta r}\log L = \sum_{i=1}^{n}\psi(z_i + r) - n\psi(r) + \sum_{i=1}^{n}\left(\frac{\sum_{j=0}^{z}\binom{z}{j}(-1)^j\left(\left(\frac{(r+j+\alpha) - 2((r+j+\alpha)+\beta)}{(r+j+\alpha)^3}\right)\right)}{\sum_{j=0}^{z}\binom{z}{j}(-1)^j\left(\frac{(r+j+\alpha)+\beta}{(r+j+\alpha)^2}\right)}\right) = 0$$

$$\frac{\delta}{\delta\alpha}\log L = \frac{2n}{\alpha} - \frac{n}{\alpha} + \sum_{i=1}^{n}\left(\frac{\sum_{j=0}^{z}\binom{z}{j}(-1)^j\left(\frac{(r+j+\alpha) - 2((r+j+\alpha)+\beta)}{(r+j+\alpha)^3}\right)}{\sum_{j=0}^{z}\binom{z}{j}(-1)^j\left(\frac{(r+j+\alpha)+\beta}{(r+j+\alpha)^2}\right)}\right) = 0$$

$$\frac{\delta}{\delta\beta} = \frac{-n}{\beta} + \sum_{i=1}^{n}\left(\frac{\sum_{j=0}^{z}\binom{z}{j}(-1)^{j}\left(\frac{1}{(r+j+\alpha)^{2}}\right)}{\sum_{j=0}^{z}\binom{z}{j}(-1)^{j}\left(\frac{(r+j+\alpha)+\beta}{(r+j+\alpha)^{2}}\right)} \right) = 0$$

The above equations can be solved through R software (3.5.2).

8. Applications of Negative Binomial Weighted Exponential Distribution

We compute the expected frequencies for fitting Poisson distribution, Negative Binomial and Negative Binomial weighted exponential distribution with the help of R studio statistical software and Pearson's chi-square test is applied to check the goodness of fit of the models discussed. The calculated figures are given in the table 1,2 and 3. Based on the chi-square, we observe that Negative Binomial weighted exponential Distribution provides a satisfactorily better fit for the data set given in table 1, 2 and3.

Table 1: Fitted proposed distribution and other competing models to a dataset representing epileptic seizure counts(see Chakraborty (2010)

epileptic seizure	**Observed counts**	**Poisson Distribution**	**NB**	**NBWED**
0	126	74.935	120.201	123.58
1	80	115.712	93.009	90.07
2	59	89.339	59.184	57.76
3	42	45.985	34.949	34.68
4	24	17.752	19.837	20
5	8	5.482	10.987	11.27
6	5	1.411	5.984	6.23
7	4	0.311	3.221	3.42
8	3	0.072	3.627	3.99
Df		4	5	7
Chi-Statistic Value		80.913	5.383	5.07
parameters		$\nu = 1.27$	$r = 1.55$ $p = 0.50$	$r = 113.72$ $\alpha = 125.75$ $\beta = 285$
p-value		0.00001	0.372	0.65

Fig 1: Graphical Overview of fitted models to Dataset representing epileptic seizure counts

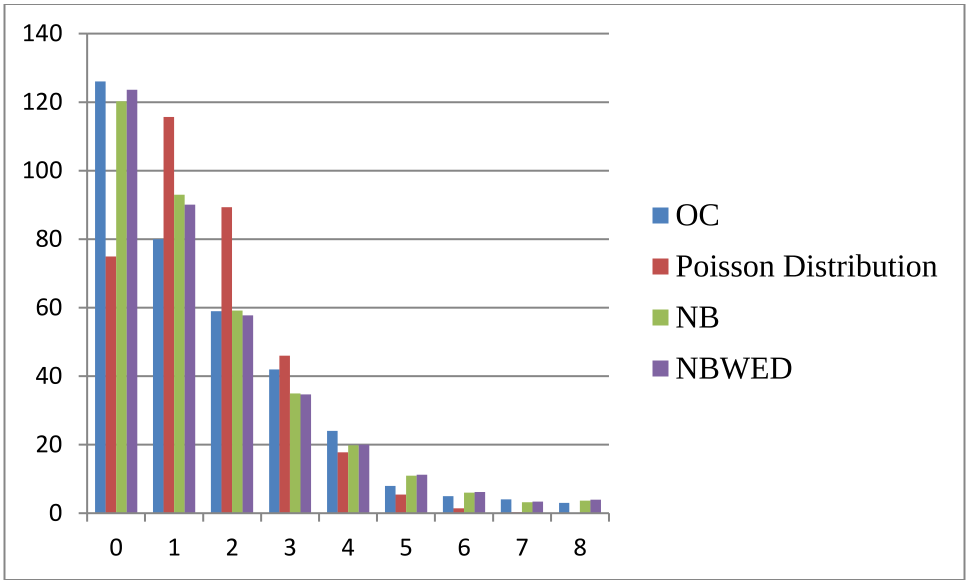


Table 2: Fitted proposed distribution and other competing models to a dataset representing number of automobile liability polices in Switzerland (see Klugmam(2004))

No. of accidents	No. of claims	Poisson	NB	NBWED
0	103704	102633.7	103723.6	103725
1	14075	15918.5	13989.9	14023.5
2	1766	1234.5	1857.1	1809.27
3	255	48	245.2	249.98
4	45	16	30	37.67
5	6	2	7	6.19
6	2	0	0.2	1.1
7	0	0	0	0
Total	119853			
Parameters		$\nu = 1.062$	$r = 0.1551$ $p = 0.15$	$r = 3.39$ $\alpha = 45$ $\beta = 6873$
Chi square		1332.3	12.12	3.49
Df		2	2	2
P-value		0	0.0023	0.17

Fig 2: Graphical Overview of fitted models to Dataset representing number of automobile liability polices in Switzerland

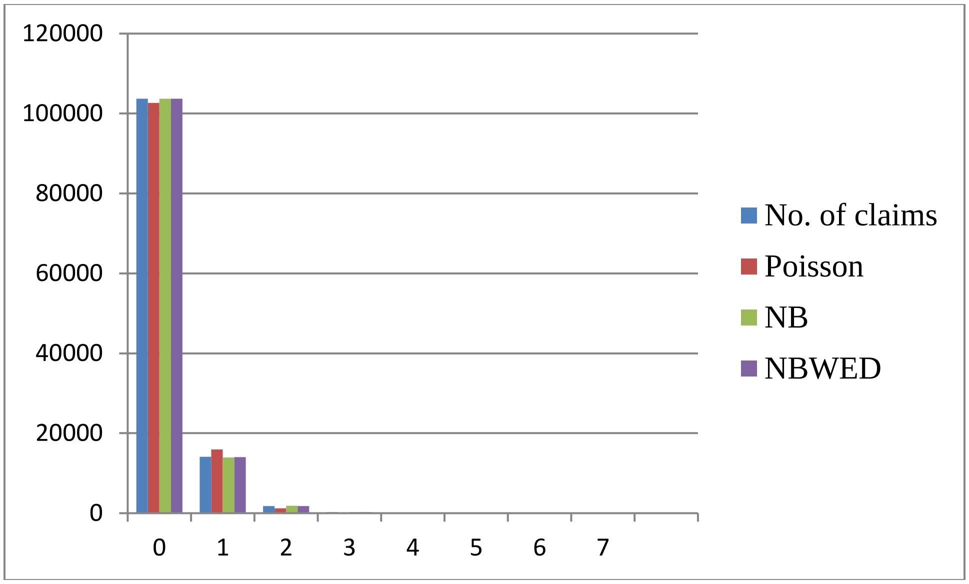
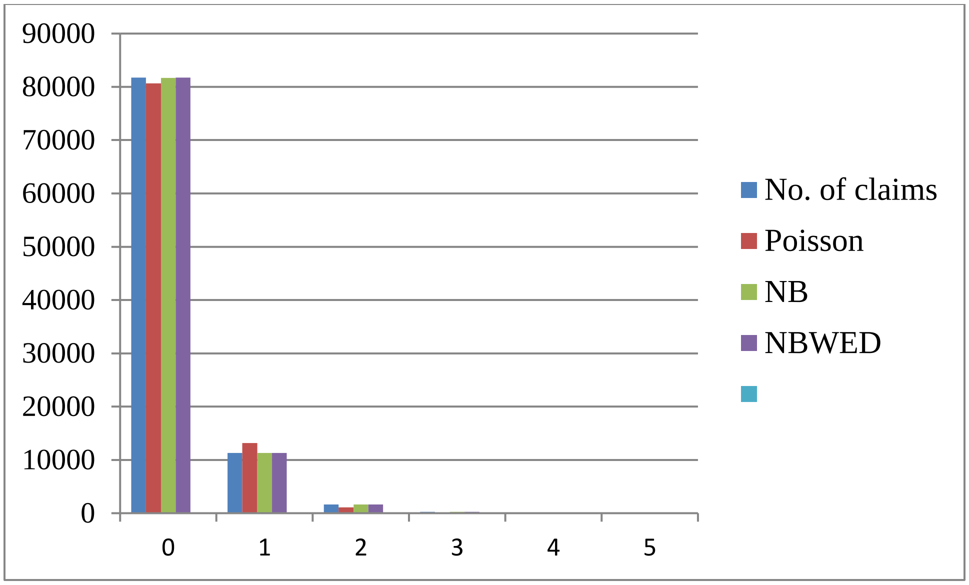


Table 3: Fitted proposed distribution and other competing models to a dataset representing number of accidents (see Klugmam(2008))

No. of accidents	No. of claims	Poisson	NB	NBWED
0	81714	80655.9	81692.6	81721.78
1	11306	13146.9	11294.8	11296.65
2	1618	1071.5	1656.2	1621.46
3	250	58.2	247.5	246.69
4	40	2.4	37.3	39.96
5	7	0.1	6.6	6.89
parameters		$\nu = 0.163$	$r = 0.89$ $p = 0.84$	$r = 6.355$ $\alpha = 57.68$ $\beta = 46.56$
Chi square		1140.7	1.14	0.06
Df		3	3	2
P-value		0	0.77	0.97

Fig 3: Graphical Overview of fitted models to Dataset representing number of accident counts



9. Conclusion: A new probability distribution is introduced using compounding technique. Statistical properties of the proposed model are studied and applied in dealing with count data.

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